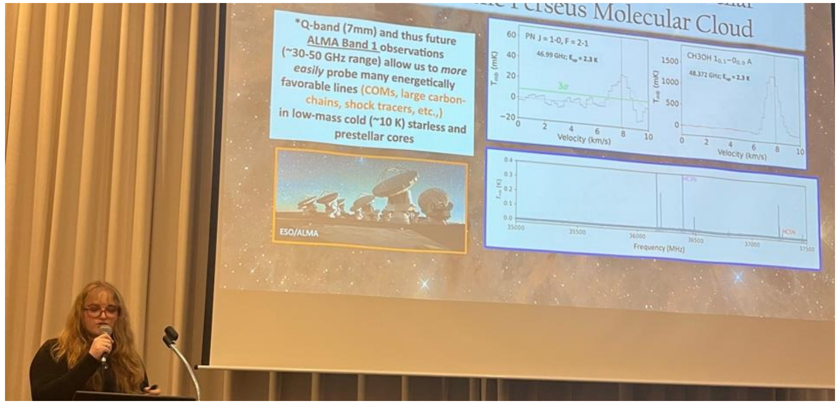
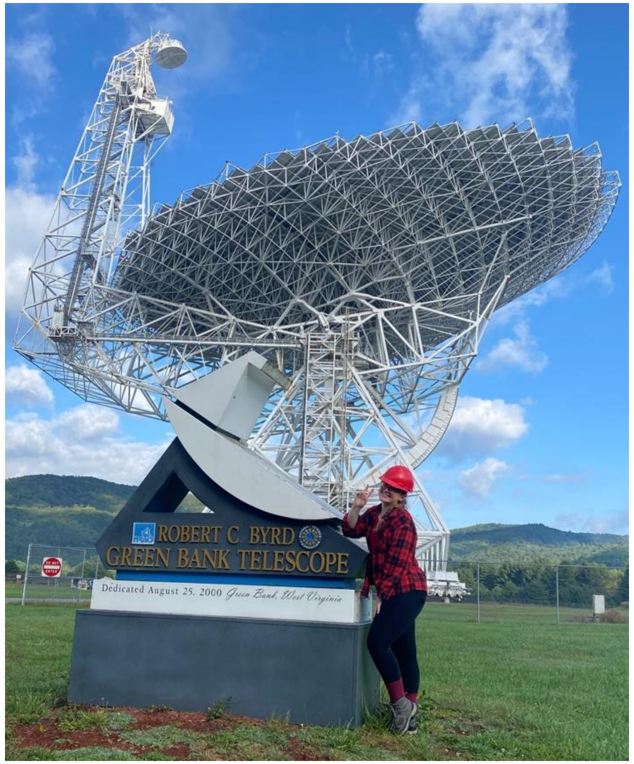
First, an Introduction 👋

Hi! My name is Samantha Scibelli, and I am in my second year as a Jansky Postdoctoral Fellow at the National Radio Astronomy Observatory (NRAO) here in Charlottesville, VA!





ASTR 5340 - Introduction to Radio Astronomy

Contact: sscibell@nrao.edu

GREEN BANK OBSERVATORY

AUI

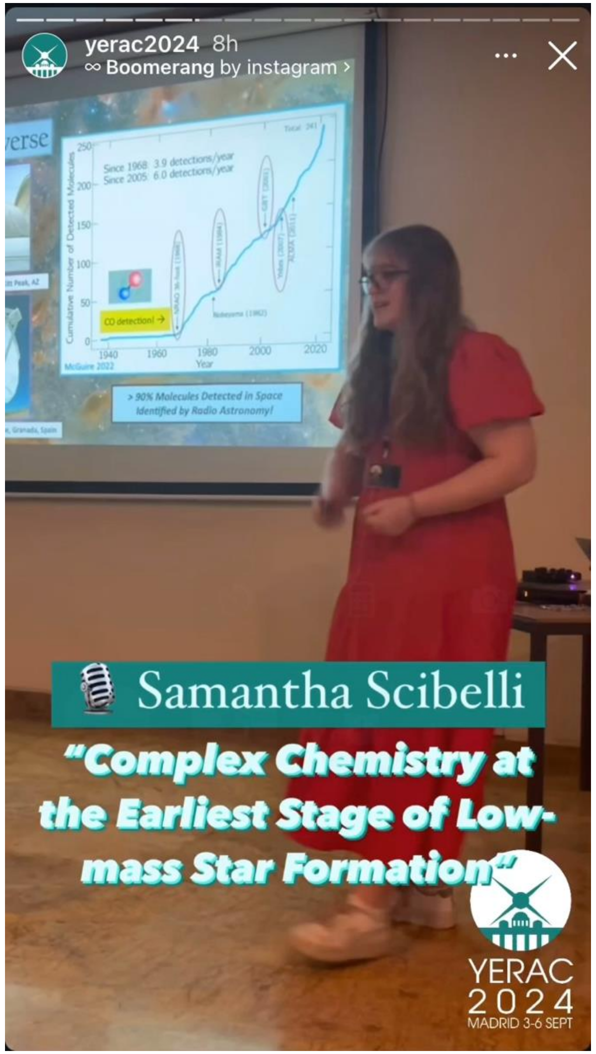
NRAO National Radio Astronomy Observatory

NSF

Where have I been?

YERAC 2024

53rd Young European Radio Astronomers Conference



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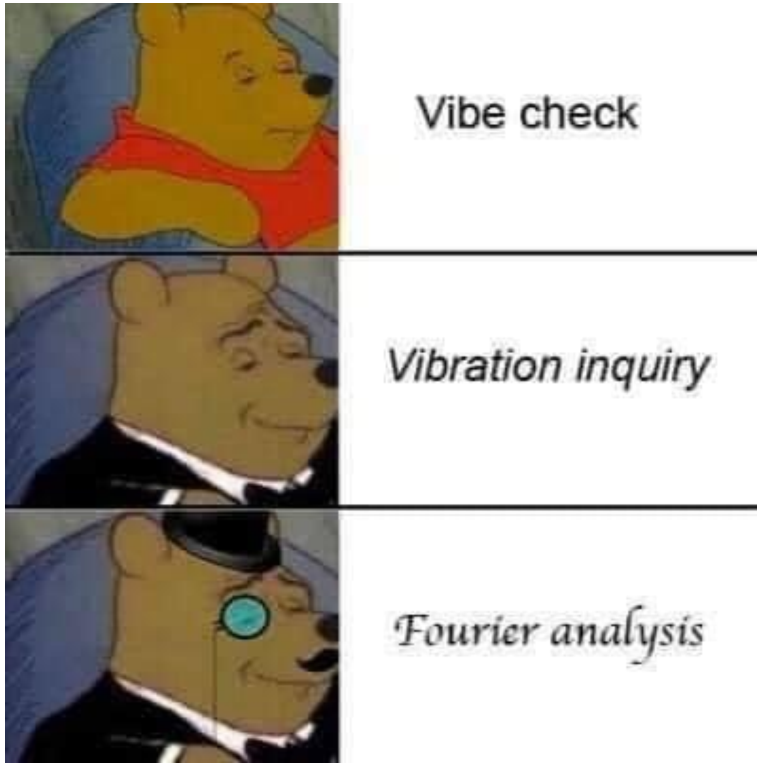
Fourier Transforms

(ERA Appendix A)



Fourier transforms are very important for radio astronomy! ***Key for signal processing, interferometry, and instruments.*** It is therefore important you familiarize and/or refresh yourself with the key properties and applications.

The Fourier transform of *f(x)* is defined by,

$$F(s) \equiv \int_{-\infty}^{\infty} f(x) \; e^{-2\pi i s x} \, dx, \tag{A.1}$$

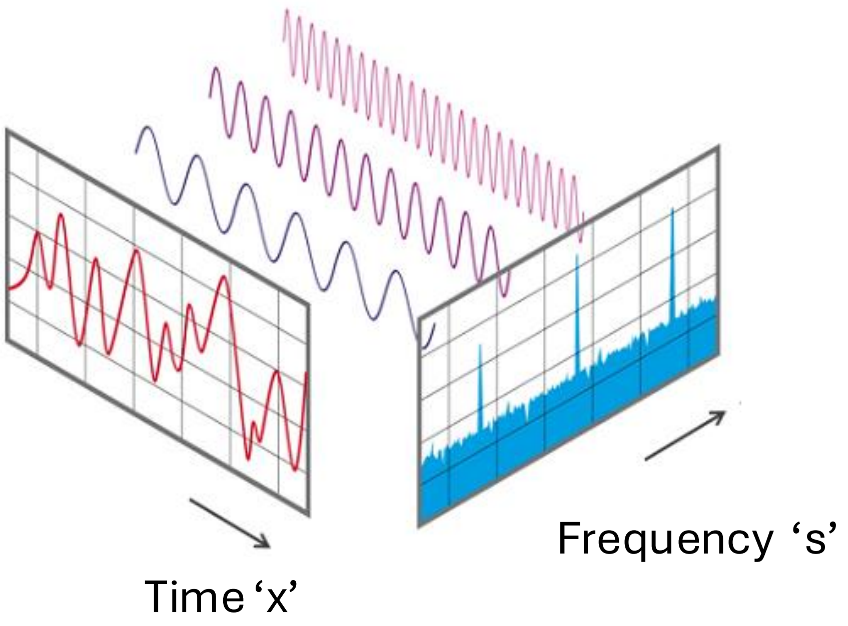
Where *f(x)* is known as the forward or inverse transform,

$$f(x) \equiv \int_{-\infty}^{\infty} F(s) \; e^{2\pi i s x} \, ds, \tag{A.2}$$

*Symbols $\rightleftharpoons$ or $\Leftrightarrow$ used to denote the "Fourier transform of…"

$$F(s) \Leftrightarrow f(x)$$

Example visual:



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- The **complex exponential** is at the heart of the transform
- Much easier to manipulate (compared to trig functions)
- Provides a **compact notation for dealing with sinusoids** of arbitrary phase
- Most physical systems we encounter obey linear differential equations represented by sinusoidal waves

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Euler's formula:

$$e^{i\phi} = \cos\phi + i\sin\phi,$$

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Let's do an example!

In astronomical data, we deal with signals discretely sampled, usually at constant intervals, and of finite duration or periodic.

↓

A **Discrete Fourier Transform (DFT)** used when only a finite number of sinusoids is needed. Usually, the DFT is computed by a **Fast Fourier Transform (FFT)** algorithm that can improve computational speeds by several orders of magnitude!

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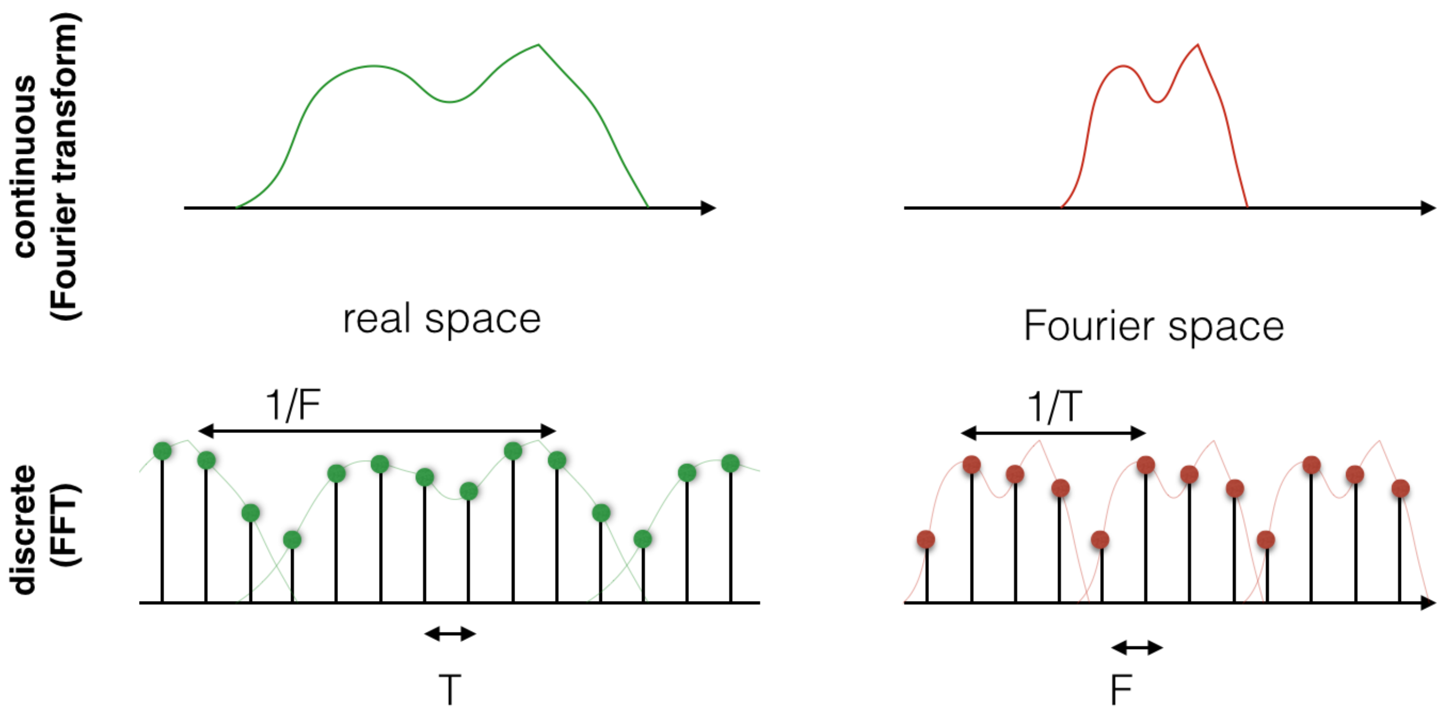
Can think of taking 'samples' from a continuous spectrum



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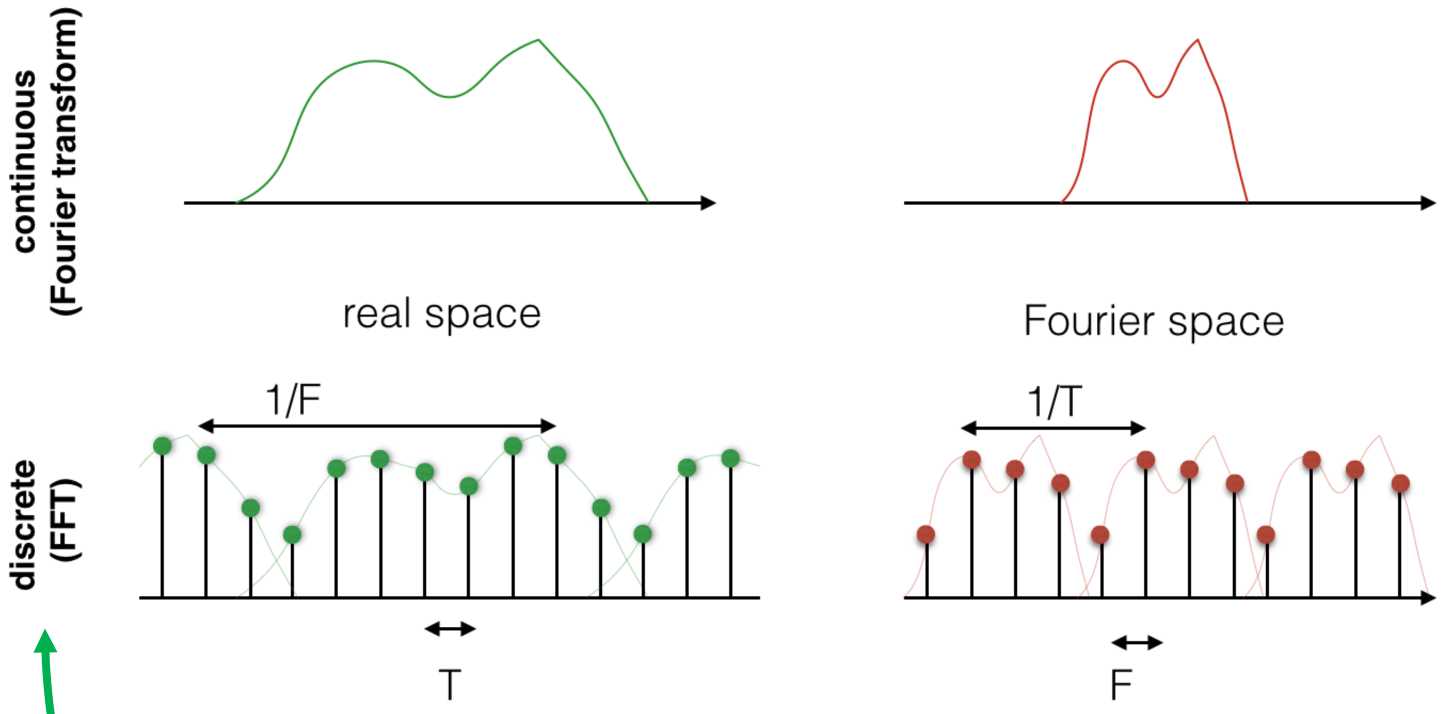

- This occurs in both the real and Fourier domains!
- For 'real' valued input this is an even function, and the imaginary part is odd where $F(s) = F^*(-s)$, i.e., DFT is **Hermitian**

Important properties of Fourier Transforms:

Addition Theorem: The Fourier transform of the sum of two functions *f(x)* and *g(x)* is the sum of their Fourier transforms *F(s)* and *G(s)*.

$$\boxed{f(x) + g(x) \Leftrightarrow F(s) + G(s)\,.} \qquad \text{(A.8)}$$

Likewise, from linearity, if a is a constant, then

$$\boxed{af(x) \Leftrightarrow aF(s)\,.} \qquad \text{(A.9)}$$

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Shift Theorem: A function *f(x)* shifted along the x-axis by a to become *f(x−a)* has the Fourier transform $e^{-2\pi ias}F(s)$. The magnitude of the transform is the same, only the phases change:

$$f(x-a) \Leftrightarrow e^{-2\pi ias}F(s)\,. \qquad \text{(A.10)}$$

Important properties of Fourier Transforms:

Similarity Theorem: For a function *f(x)* with a Fourier transform F(s), if the x-axis is scaled by a constant a so that we have *f(ax)*, the Fourier transform becomes $|a|^{-1}F(s/a)$.

$$f(ax) \Leftrightarrow \frac{F(s/a)}{|a|}. \qquad \text{(A.11)}$$

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Modulation Theorem: Very important in radio astronomy as it describes how signals can be "mixed" to different intermediate frequencies (IFs):

$$f(x)\cos(2\pi\nu x) \Leftrightarrow \frac{1}{2}F(s-\nu) + \frac{1}{2}F(s+\nu). \qquad \text{(A.12)}$$

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Let’s look at online tool!

https://www.falstad.com/fft/index.html

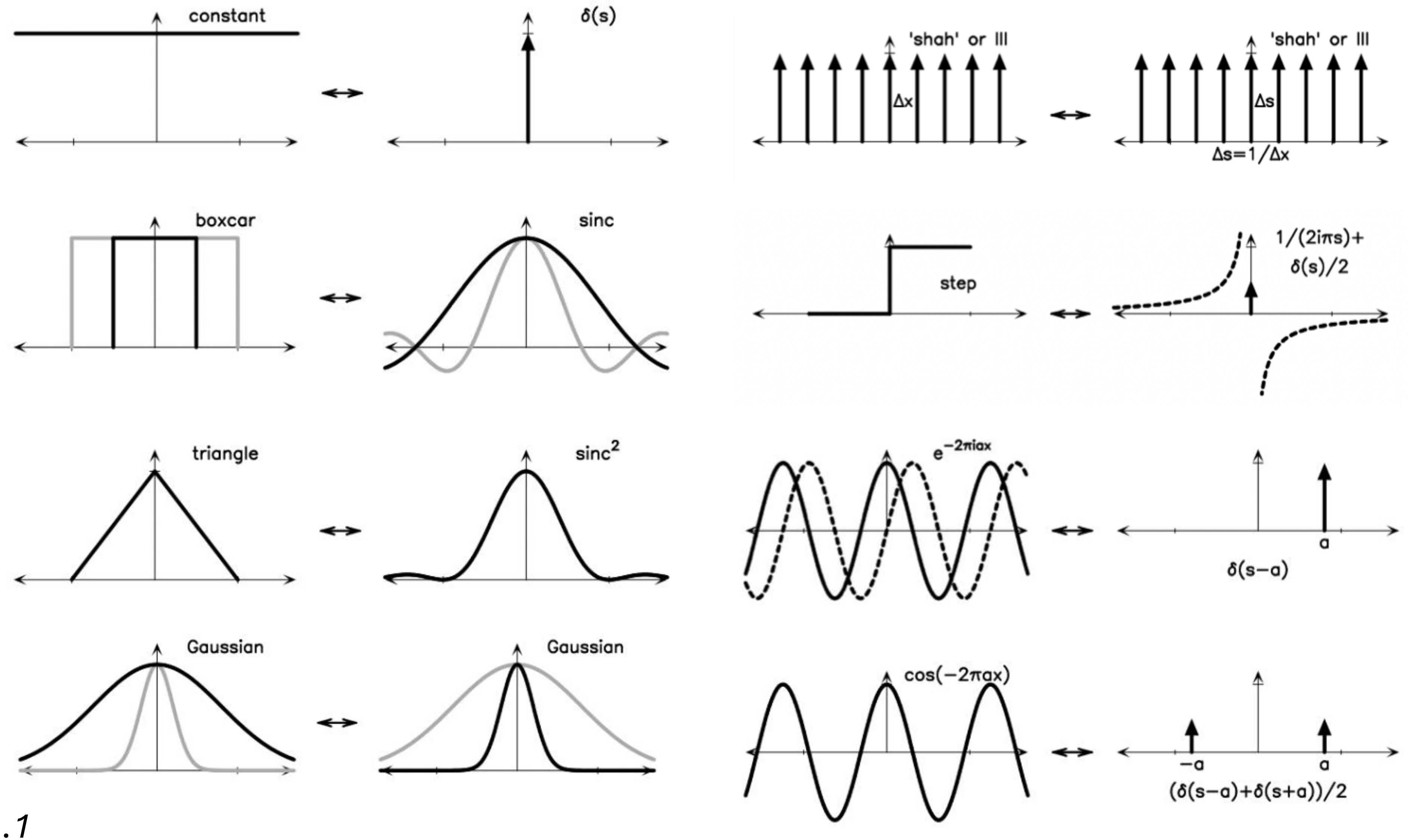
constant
δ(s)
'shah' or III
Δx
'shah' or III
Δs
Δs=1/Δx
boxcar
sinc
step
1/(2iπs)+
δ(s)/2
triangle
sinc²
e^{−2πiax}
a
δ(s−a)
Gaussian
Gaussian
cos(−2πax)
−a
a
(δ(s−a)+δ(s+a))/2

Fig. A.1

Important applications of Fourier Transforms:

Convolution multiplies one function f by the time reversed **kernel** function g, shifts g by some amount u, and integrates u from −∞ to +∞

$$h(x) = f * g \equiv \int_{-\infty}^{\infty} f(u)\, g(x-u)\, du. \quad \text{(A.14)}$$

Convolution theorem:

$$f * g \Leftrightarrow F \cdot G. \quad \text{(A.15)}$$

Example:

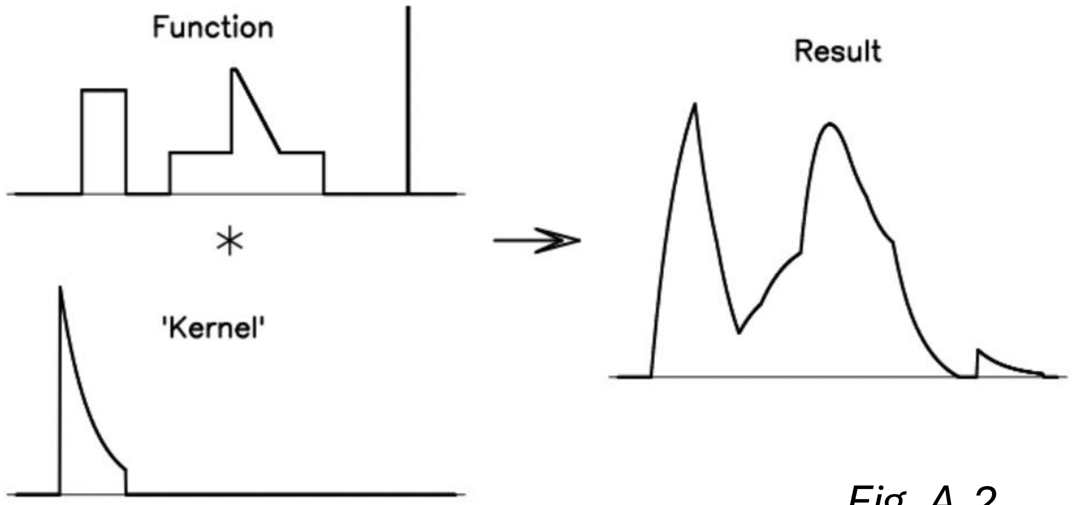

Fig. A.2

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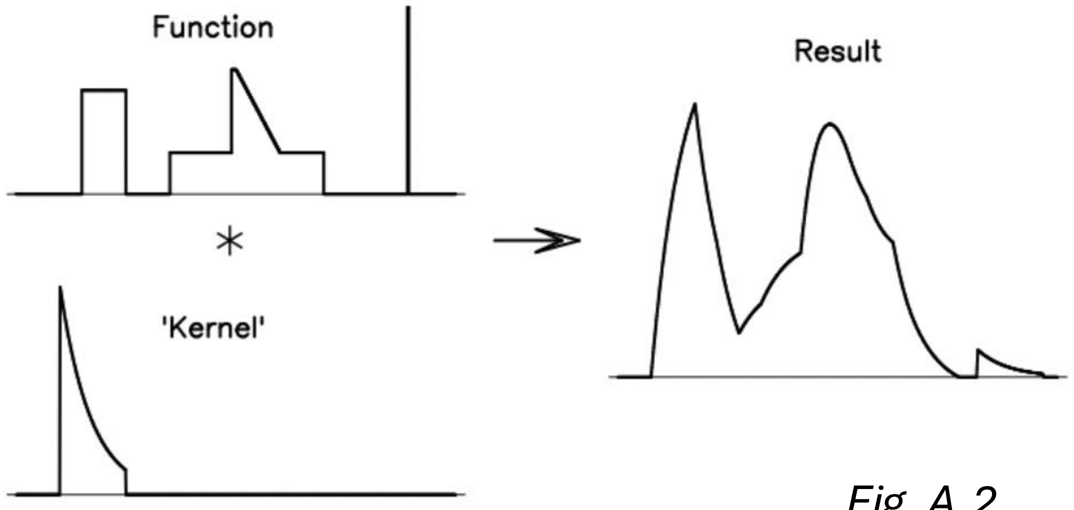

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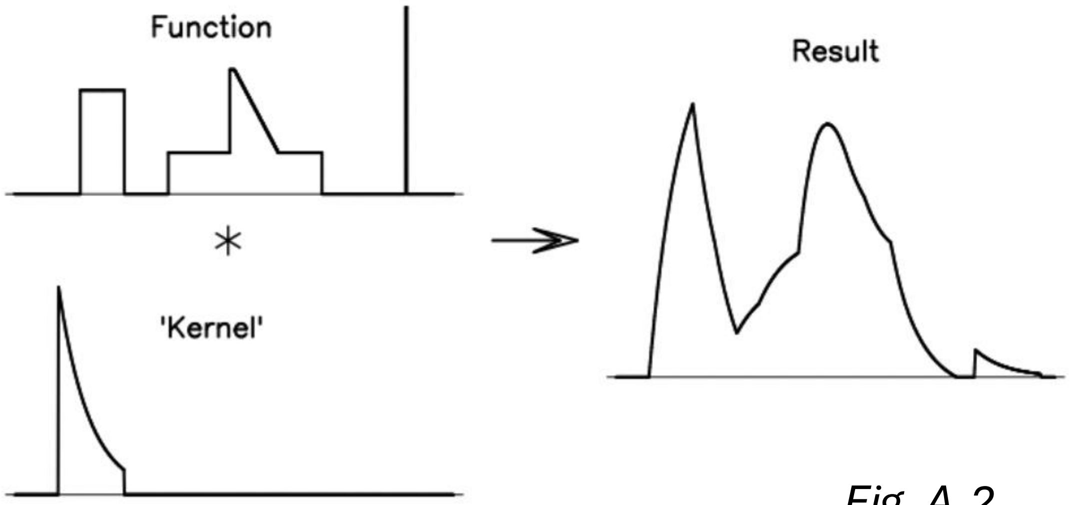

Fig. A.2

Let's look at online tool!

https://phiresky.github.io/convolution-demo/

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Cross-correlation is a very similar operation to convolution, except that the kernel is **not time-reversed**:

$$f \star g \equiv \int_{-\infty}^{\infty} f(u)\, g(u-x) \ \ du. \qquad \text{(A.16)}$$

Cross-correlation theorem:

$$f \star g \Leftrightarrow \overline{F} \cdot G. \qquad \text{(A.17)}$$

NOTE: unlike for convolution,

$$f(x) \star g(x) \neq g(x) \star f(x)$$

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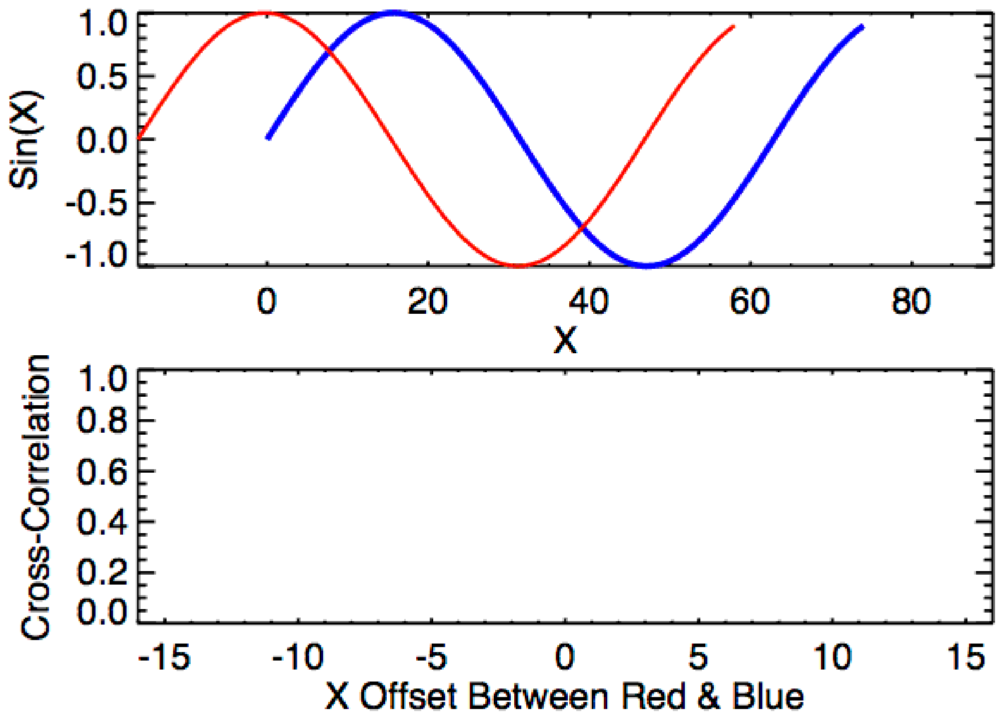

AKA How similar are your two signals?

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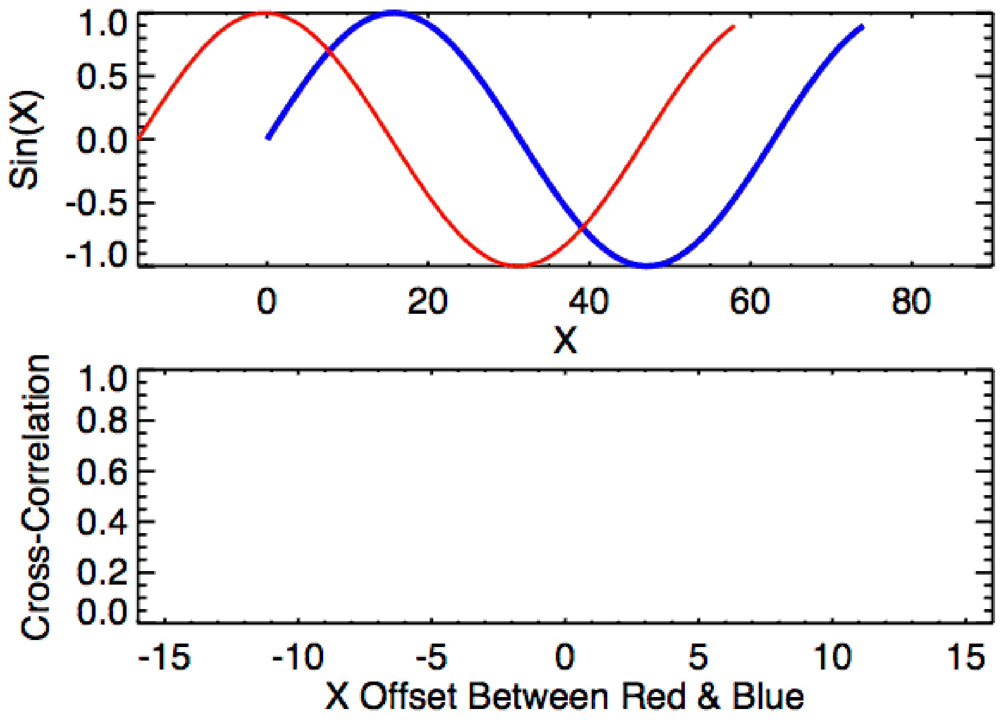

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Autocorrelation is a special case (Wiener-Khinchin theorem):

$$f \star f \Leftrightarrow \overline{F} \cdot F = |F|^2 \qquad \text{(A.18)}$$

Important applications of Fourier Transforms:

$$\begin{array}{ccc} x_j & \Longleftrightarrow & X_k \\ \text{(function)} & \text{DFT} & \text{(transform)} \\ \Downarrow & & \Downarrow \\ x_j \star x_j & \Longleftrightarrow & |X_k|^2 \\ \text{(autocorrelation)} & \text{DFT} & \text{(power spectrum)} \end{array}$$

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Show and tell time! Pass around correlator card from the obsolete Arizona Radio Observatory (ARO) Millimeter Autocorrelator (MAC) backend spectrometer

Important applications of Fourier Transforms:

****<u>Sampling Theorem</u> or Nyquist-Shannon Theorem**: any bandwidth-limited (or **band-limited**) continuous function confined within the frequency range Δv may be reconstructed ***exactly*** from uniformly spaced samples separated in time by $\leq (2\Delta v)^{-1}$. The critical sampling rate $(\Delta t)^{-1} = 2\Delta v$ is known as the **Nyquist rate**, and the spacing between samples must satisfy $\Delta t \leq 1/(2\Delta v)$ seconds

Aliasing ex:

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The frequency of the sampled bandwidth is the **Nyquist frequency:**

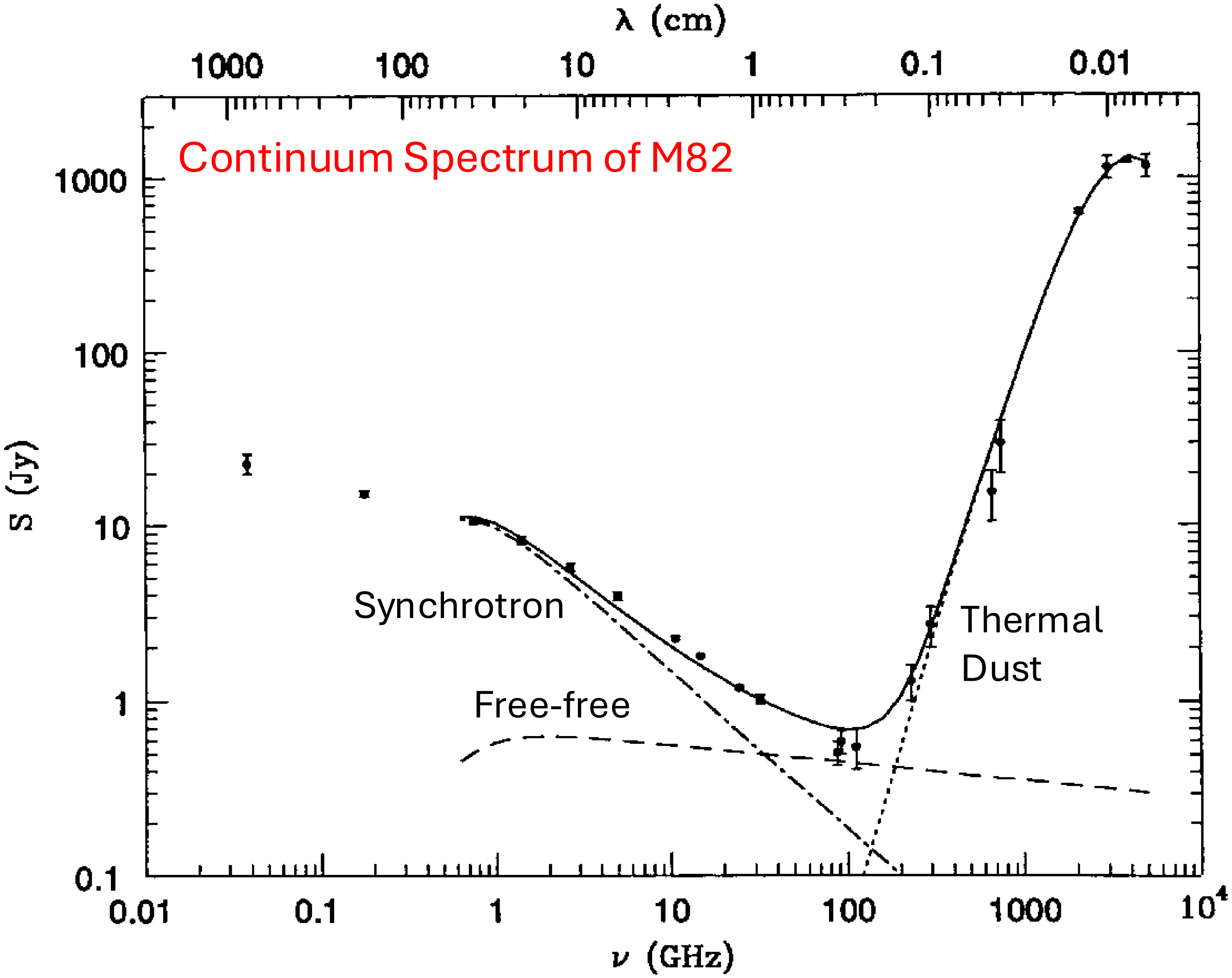
$$\nu_{N/2} = 1/(2\,\Delta t)\,. \quad \text{(A.6)}$$

More Dust Emission (It's so important!)

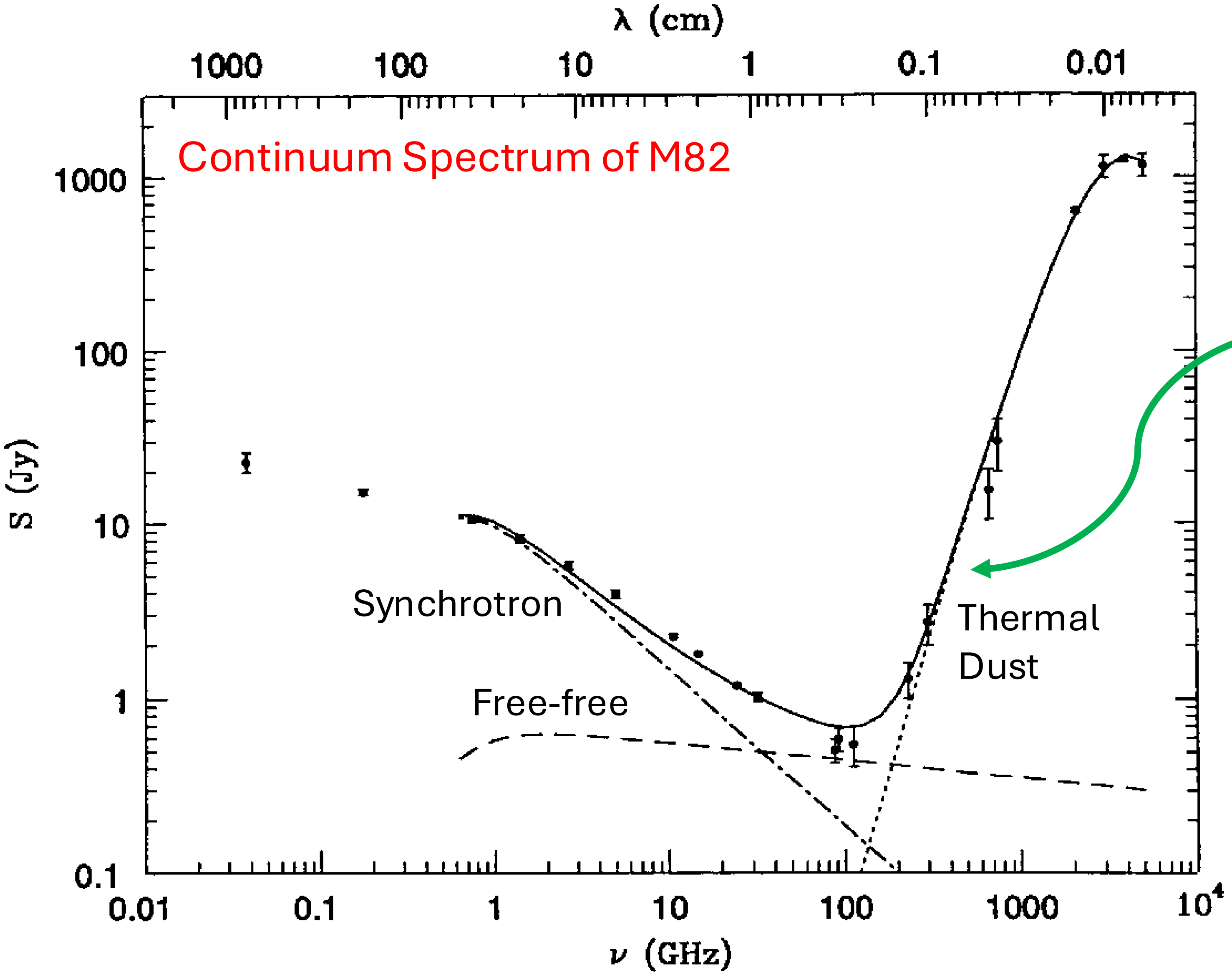
(ERA Chap. 2.8 + THz Astronomy Chap. 3)

FUN FACT!

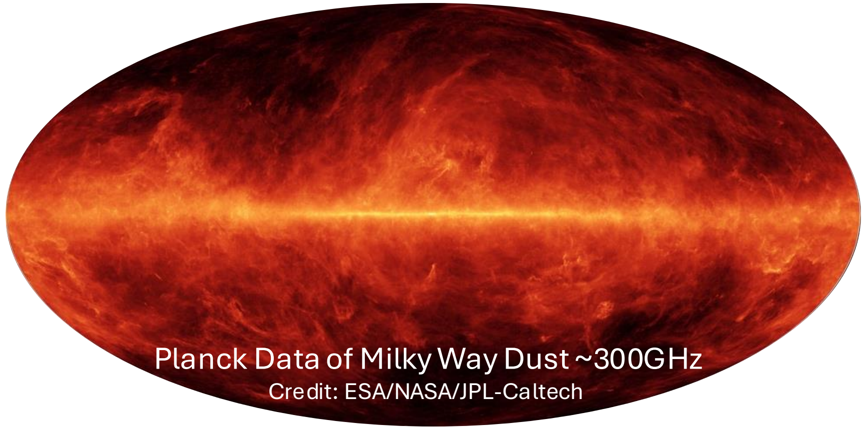
Dust grains account for only ~1% of the ISM (interstellar medium) by mass but are responsible for absorbing ~30% of the Universe's starlight and reradiating it at THz frequencies!



λ (cm)
1000
100
10
1
0.1
0.01
Continuum Spectrum of M82
1000
100
10
1
0.1
S (Jy)
Synchrotron
Free-free
Thermal
Dust
0.01
0.1
1
10
100
1000
10^4
ν (GHz)



- Thermal dust emission dominates over other forms of continuum radiation beyond around 300 GHz (or ~1mm)!
- This is due to wide-spread existence of ~12-30 K dust and the wavelength dependence of the emission mechanisms


Planck Data of Milky Way Dust ~300GHz
Credit: ESA/NASA/JPL-Caltech

Key Concept: **Extinction**

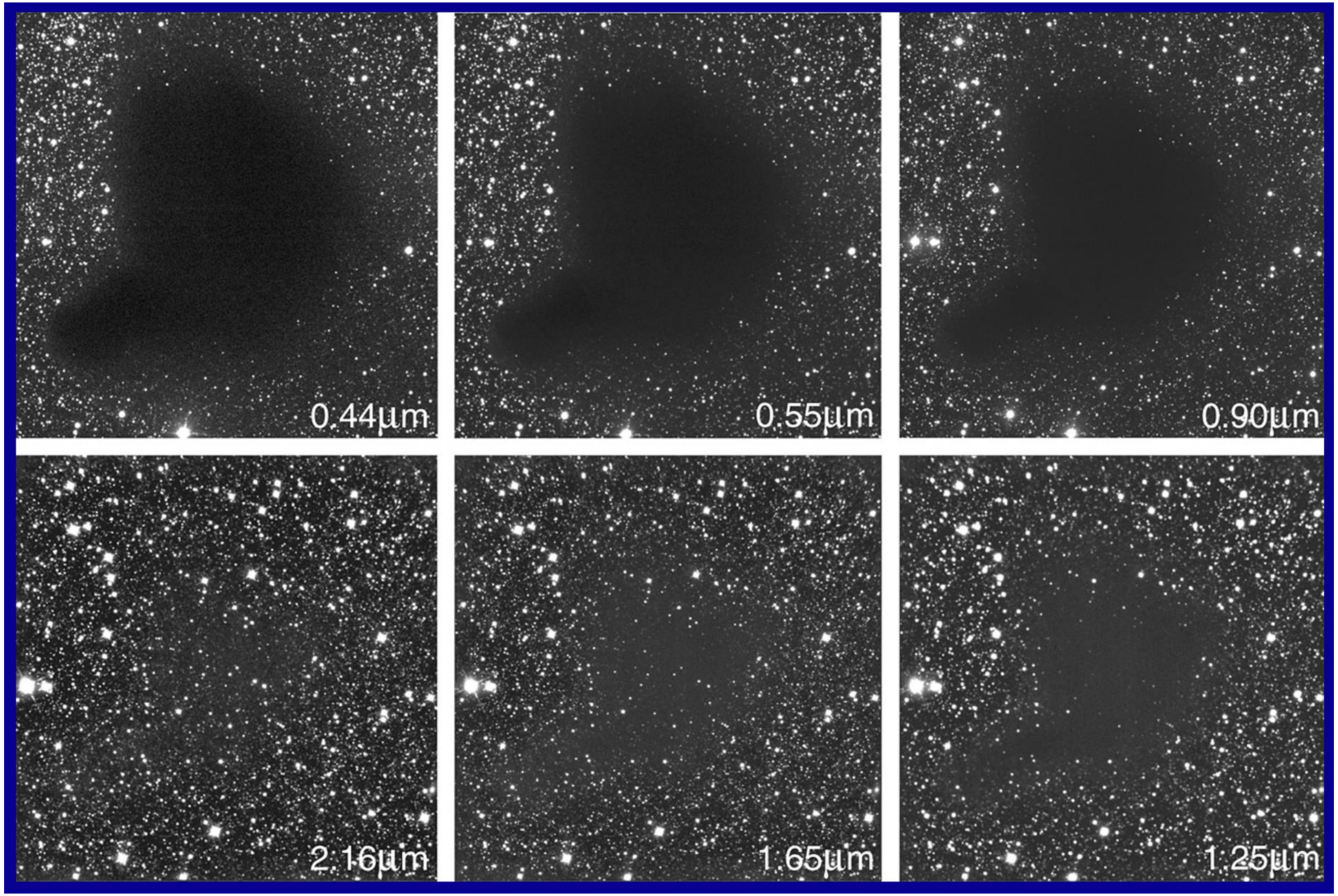
Extinction = Absorption + Scattering

Your equation of transfer now includes an **'extinction coefficient' or dust opacity** that includes an extinction emissivity factor, Q_{ext}, that is **dependent on wavelength** where,

$$\kappa_\lambda = n_d Q_e(\lambda) \sigma_d \quad [\mathrm{cm}^{-1}]$$

n_d: Particle number density [cm^{-3}}

σ_d: Cross section [cm^2]



The extinction, A_ν, of starlight by dust is defined in terms of **magnitudes** of extinction,

$$A_\nu = -2.5\log\left|\frac{S_\nu^o}{S_\nu^i}\right| = 1.086\tau_D = 1.086 N_D Q_{ext}(\pi a^2)$$

(3.1 *in THz Astronomy*)

where,

S_ν^o = observed flux density (Jy)
S_ν^i = intrinsic flux density (Jy)
τ_D = dust optical depth
a = dust grain radius (cm)
N_D = dust column density (cm^{-2})
Q_ν^{ext} = emissivity of dust grain, that is, how much like a black body it appears to be;
1 $\Leftrightarrow$ true black body, 0 $\Leftrightarrow$ fully invisible

Remember this is a combination of scattering and absorption (and it also depends on wavelength or frequency)!

$$Q_{ext} = Q_{abs} + Q_{scat}$$

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$$\frac{N_H}{A_\nu} \approx 1.79 \times 10^{21} \frac{\text{atoms}}{\text{cm}^2\,\text{mag}}$$

(3.3 *in THz Astronomy*)

(Predehl and Schmitt, 1995)

Another really cool thing… **Extinction curves** provide clues to the **composition and size distribution of dust grains**!

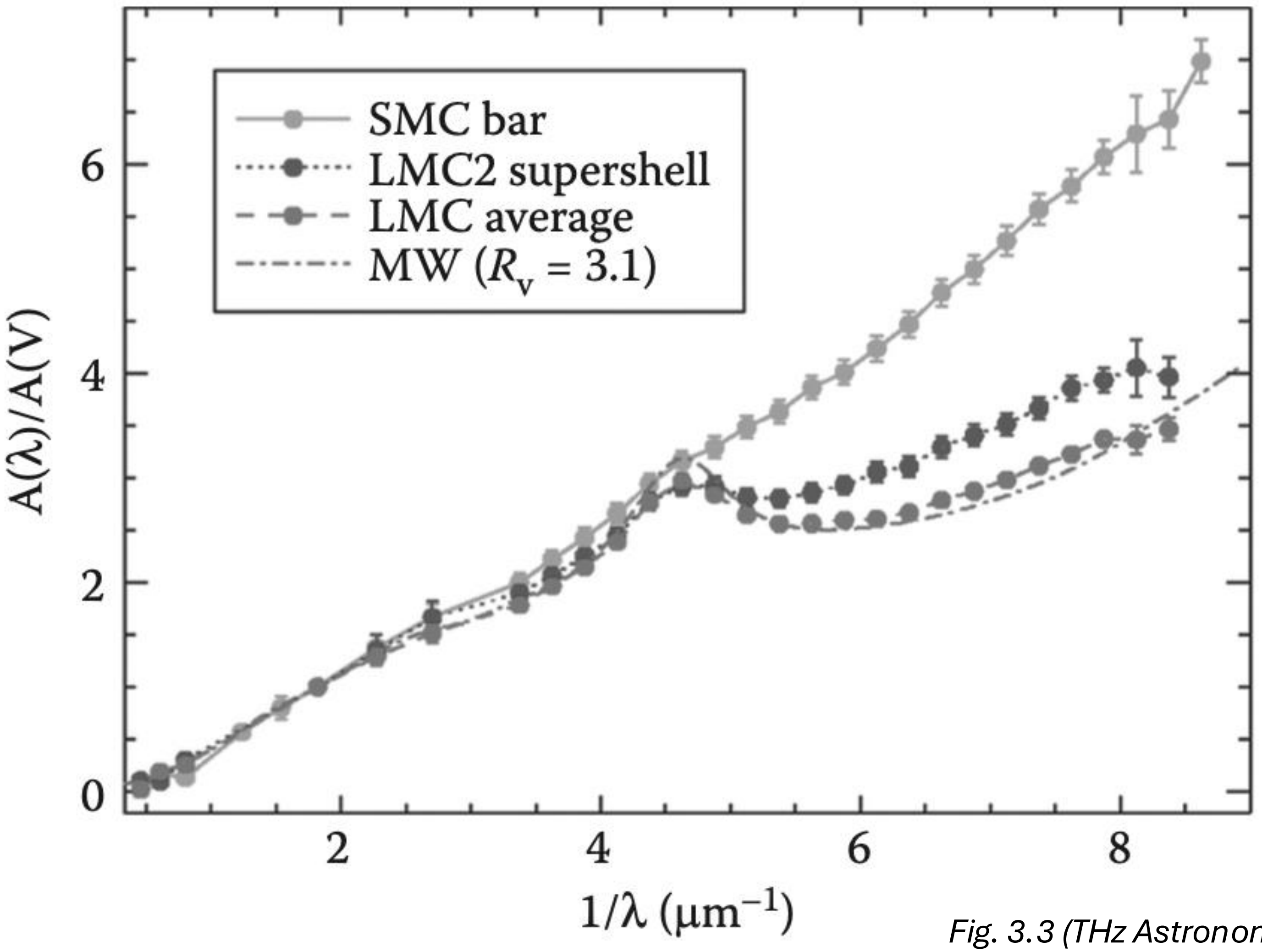


Fig. 3.3 (THz Astronomy)

Another really cool thing… **Extinction curves** provide clues to the **composition and size distribution of dust grains**!

- Grains typically made up of graphite and silicates (the major component), plus polycyclic aromatic hydrocarbons (PAHs)
- Individual grain radii extend from less than or equal to 0.005 (PAHs) to ~0.12 microns (silicate and carbonaceous)
- BUT! Grains grow and accumulate on ice mantles and can reach ~0.1mm to mm sizes! **This changes the effective opacity and can affect measurements of temperature, density, mass, etc.,!**

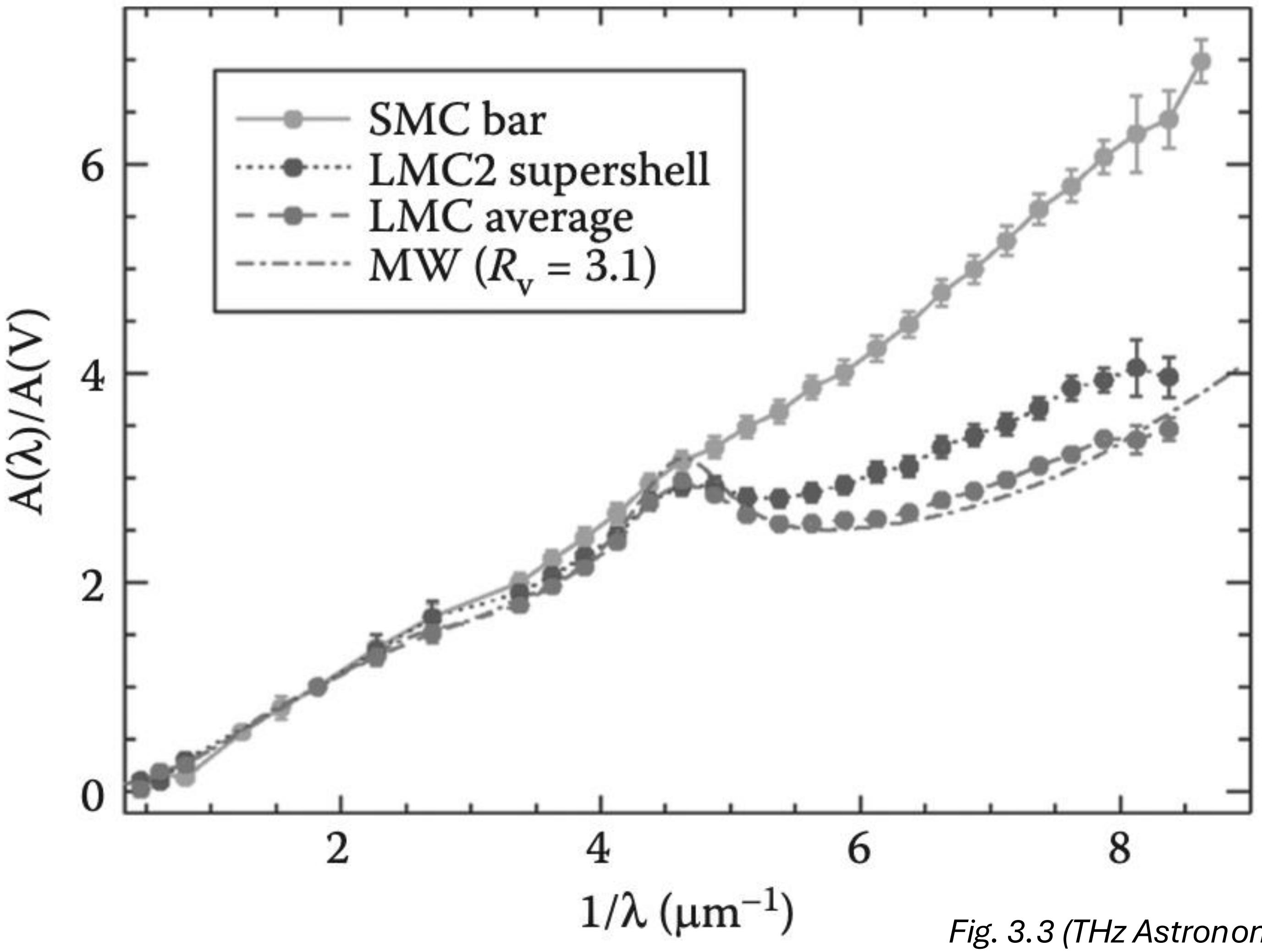


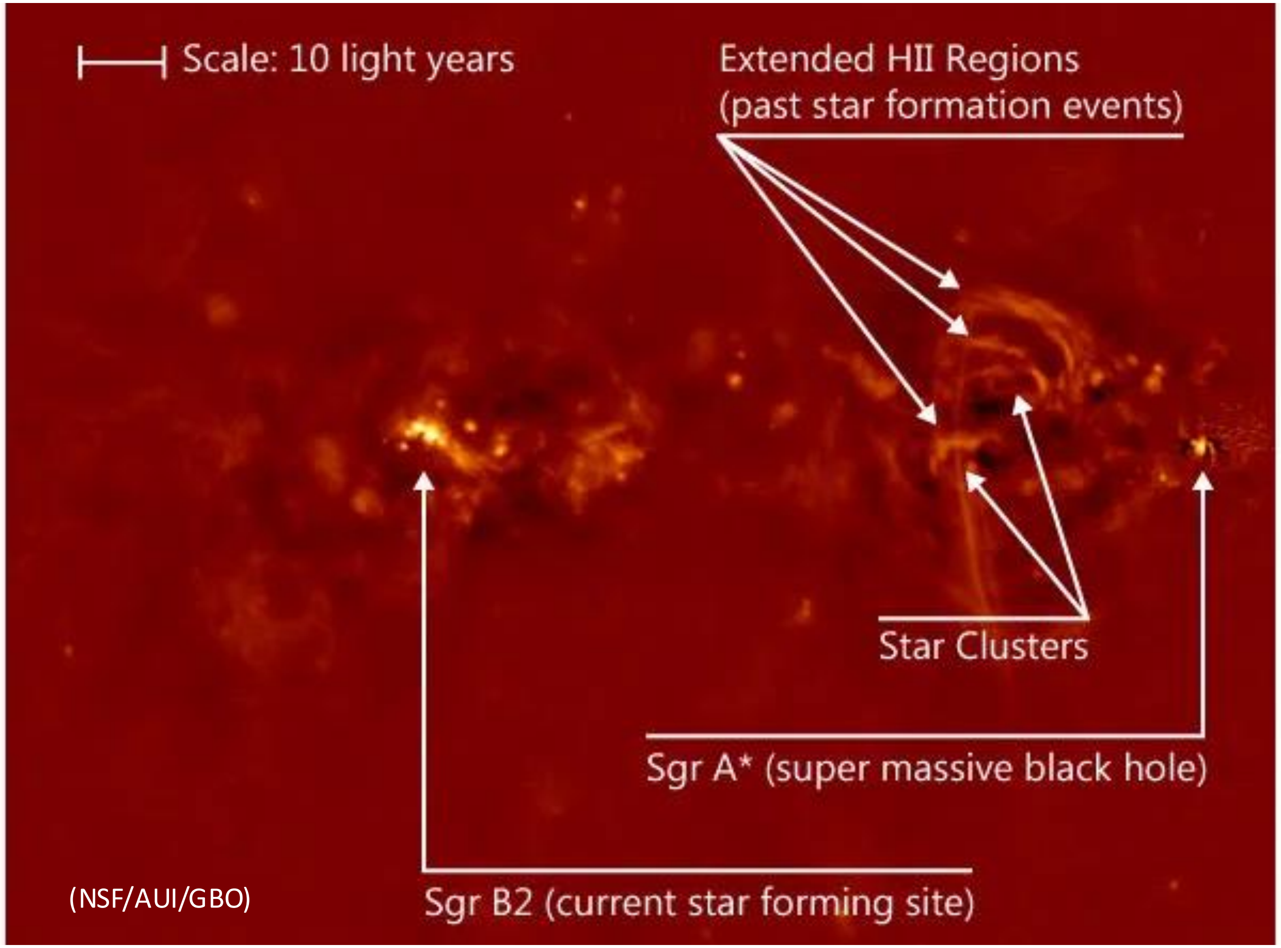
Fig. 3.3 (THz Astronomy)

Dust Emission in practice

Keep in mind for Final Project!

Recent results from the **Green Bank Observatory**'s Continuum Instrument, MUSTANG-2, that observes the sky at wavelengths of 3mm (90 GHz)

Image shows inner ~ 300 light years → of Galactic Center.
Look how dynamic and dusty it is!



Ginsburg et al., 2020

Dust Emission in practice

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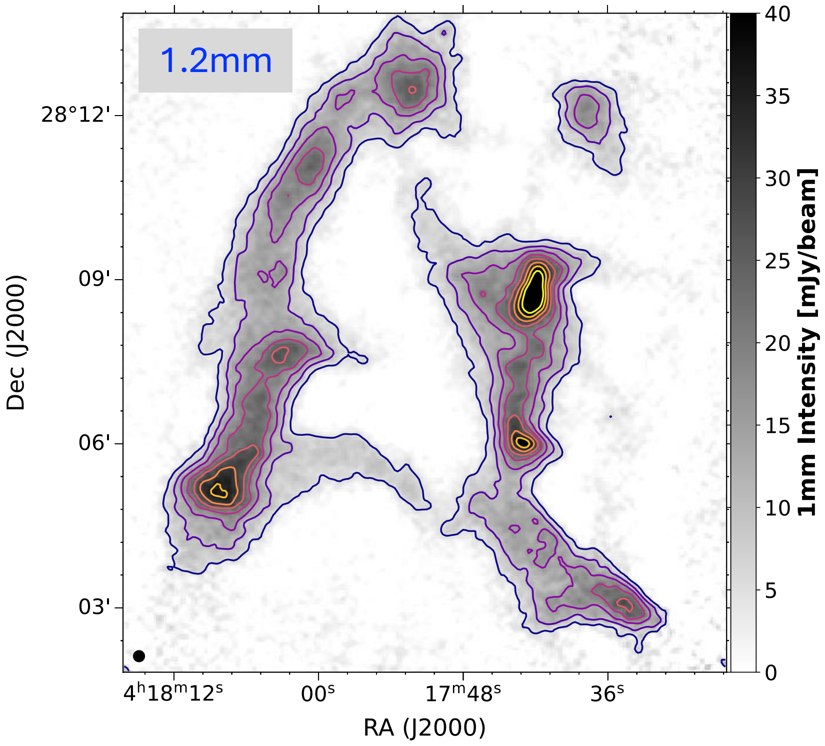
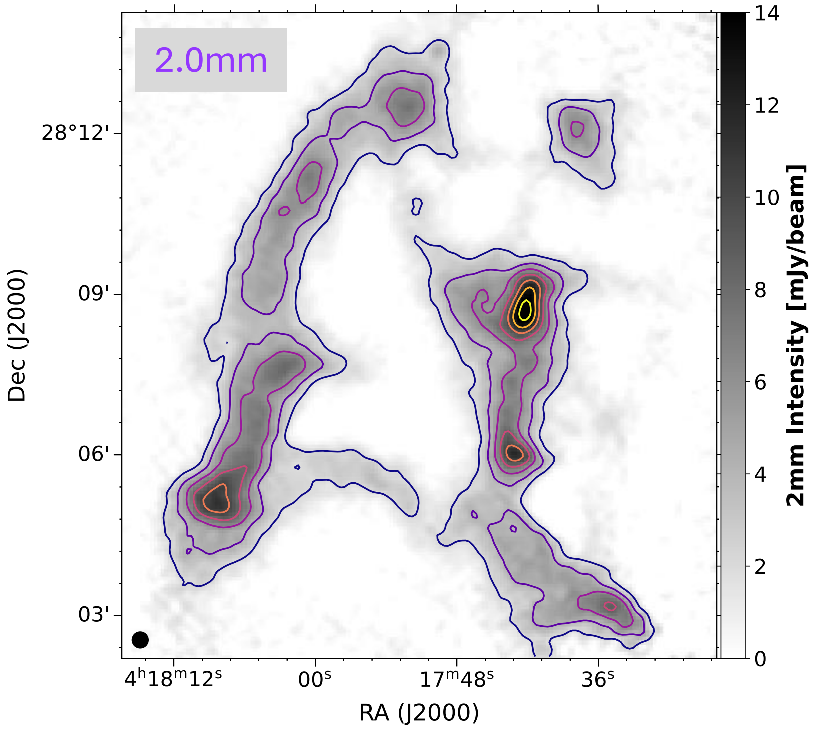
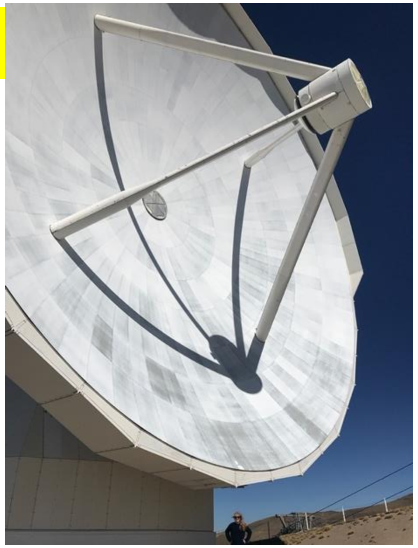
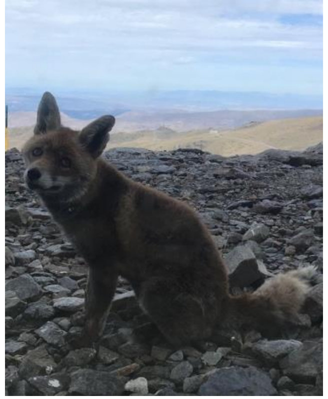
Data Taken with NIKA2 Instrument on IRAM 30m →









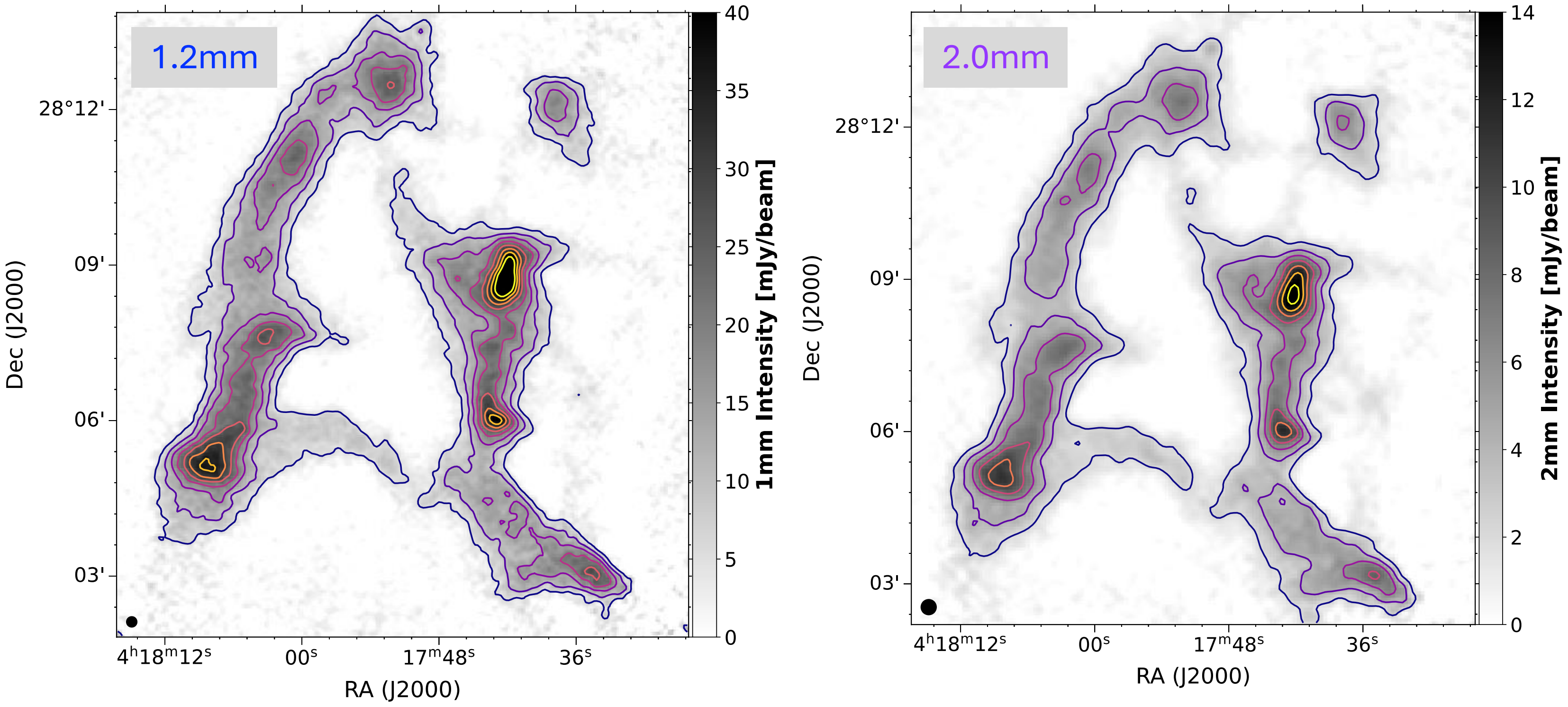
Scibelli et al., 2023

Dust Emission in practice

Keep in mind for Final Project!

Dust opacity, κ_ν, can be directly probed with these two maps! Starting with relation →

$$\kappa_\nu = \kappa_0 \left(\frac{\nu}{\nu_0}\right)^\beta$$



Scibelli et al., 2023

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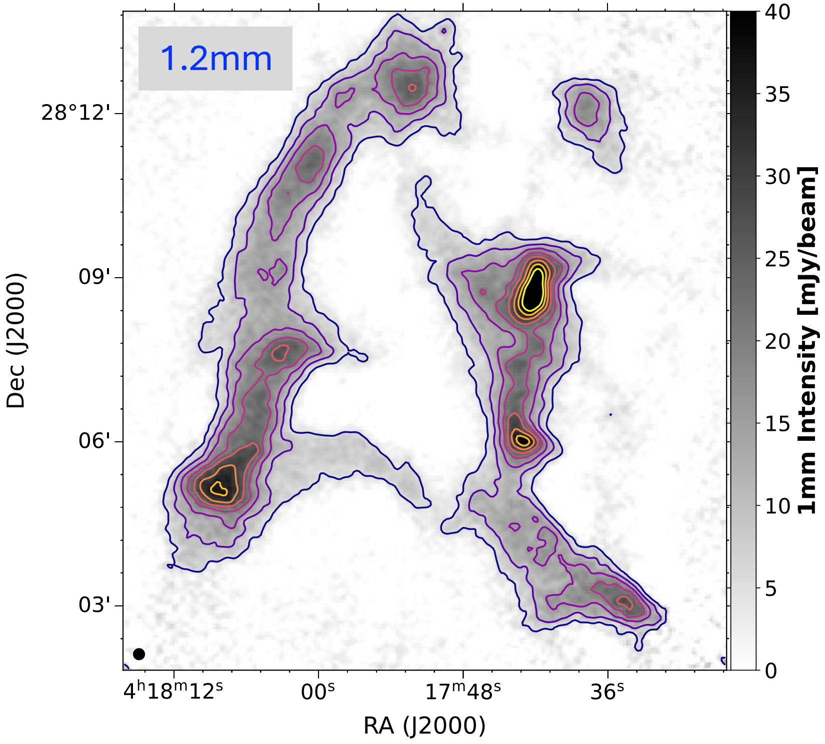

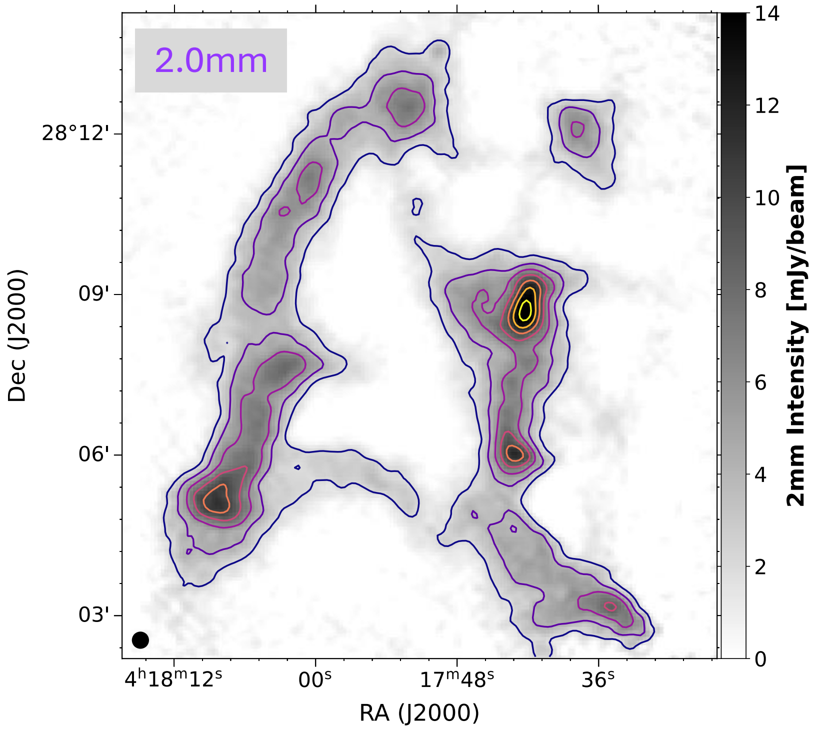

Where, in the optically thin limit,

$$S_\nu = B_\nu[T_d]\Omega\tau_\nu,$$

For,

$$\tau_\nu = \kappa_\nu \Sigma$$

Scibelli et al., 2023

Dust Emission in practice

Keep in mind for Final Project!

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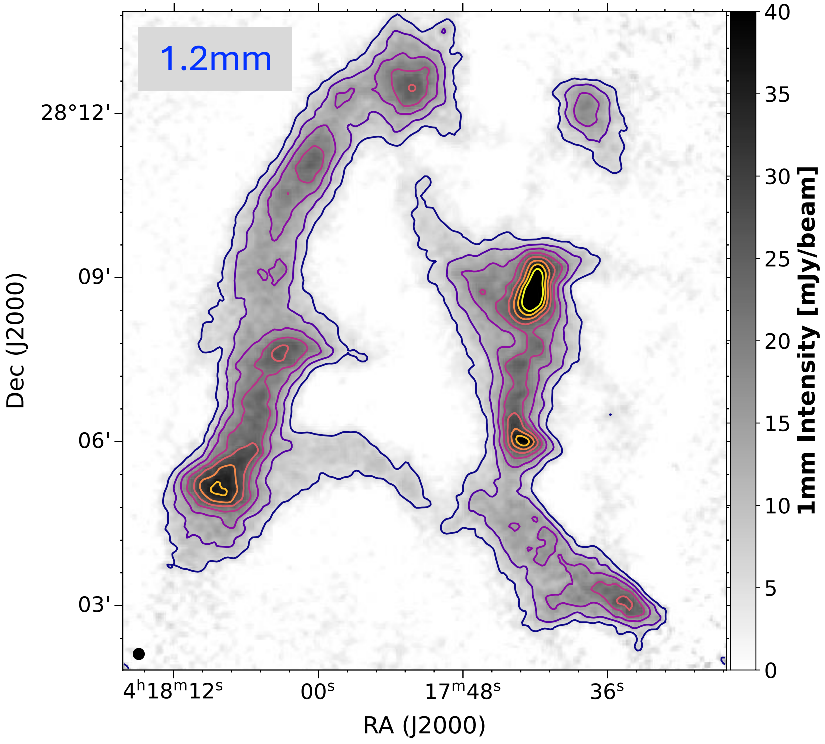

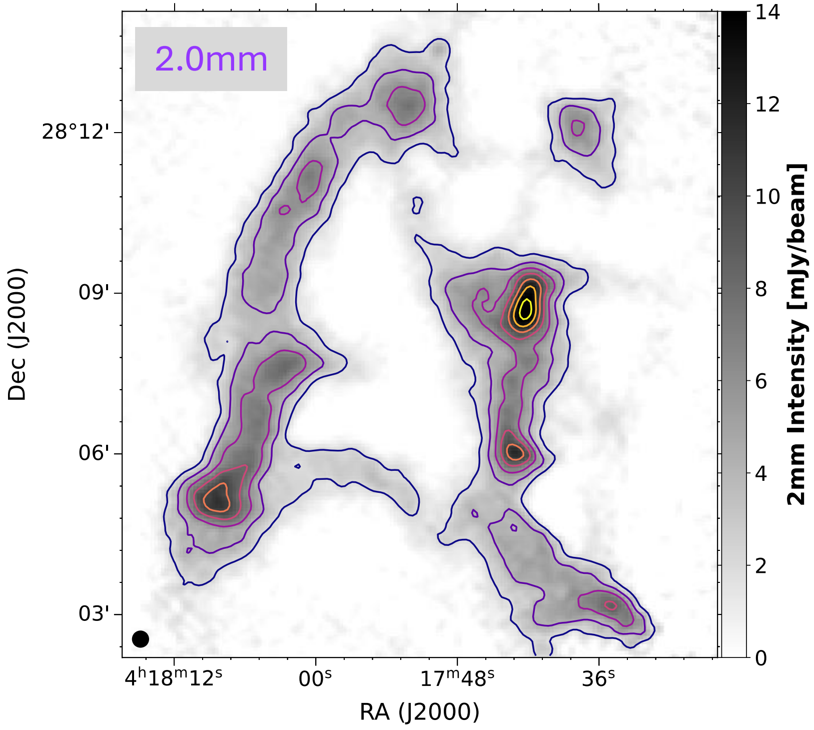

Scibelli et al., 2023

Where, in the optically thin limit,

$$S_\nu = B_\nu[T_d]\Omega\tau_\nu,$$

For,

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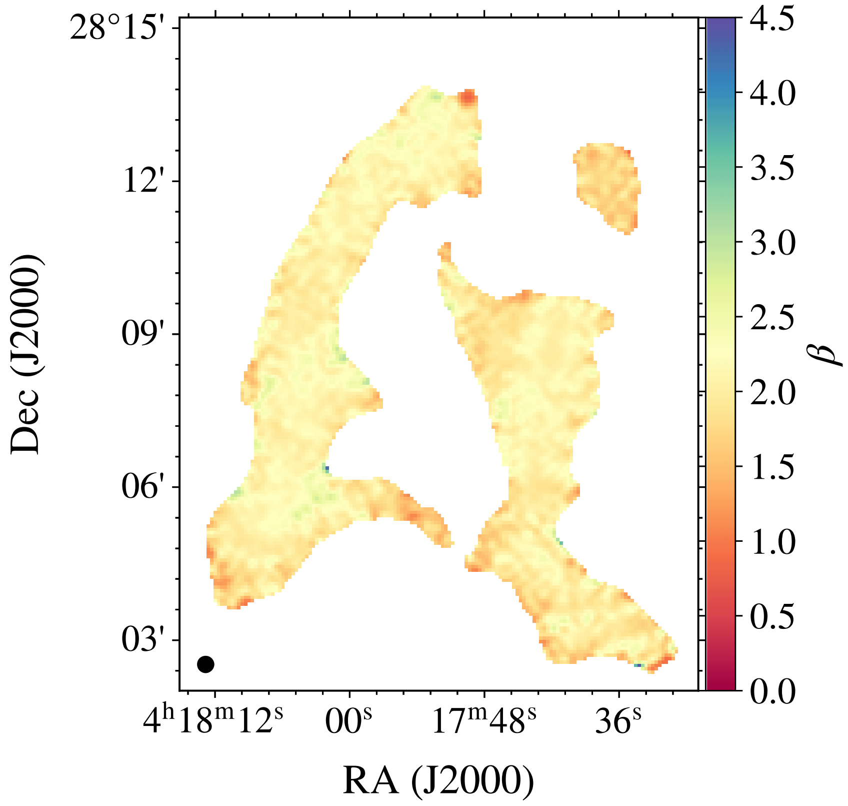
You can exploit this to find β using the **ratio of the two maps**!

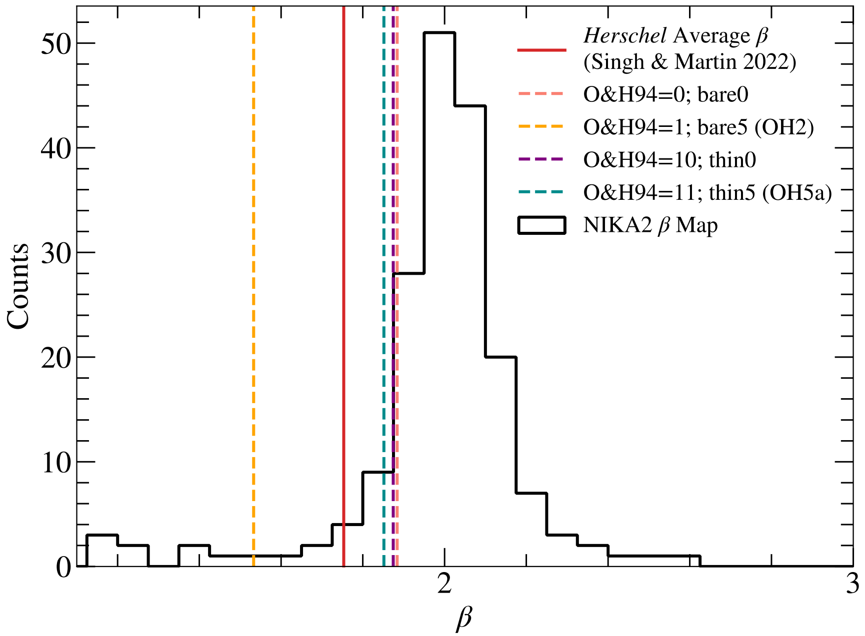
$$R_{1,2} = \frac{B_{\nu_{1.2mm}}[T_d]}{B_{\nu_{2.0mm}}[T_d]} \left(\frac{\nu_{1.2mm}}{\nu_{2.0mm}}\right)^\beta$$

$$\beta = \frac{\log(R_{1,2} \times B_{\nu_{2.0mm}}[T_d]]/B_{\nu_{1.2mm}}[T_d])}{\log(\nu_{1.2mm}/\nu_{2.0mm})}$$

Dust Emission in practice

Keep in mind for Final Project!





Scibelli et al., 2023

"This suggests that the **opacity laws** we use in our radiative transfer analysis, which are also used elsewhere throughout the literature, **do not accurately describe the observed** $\boldsymbol{\beta}$"

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Dust Emission in practice

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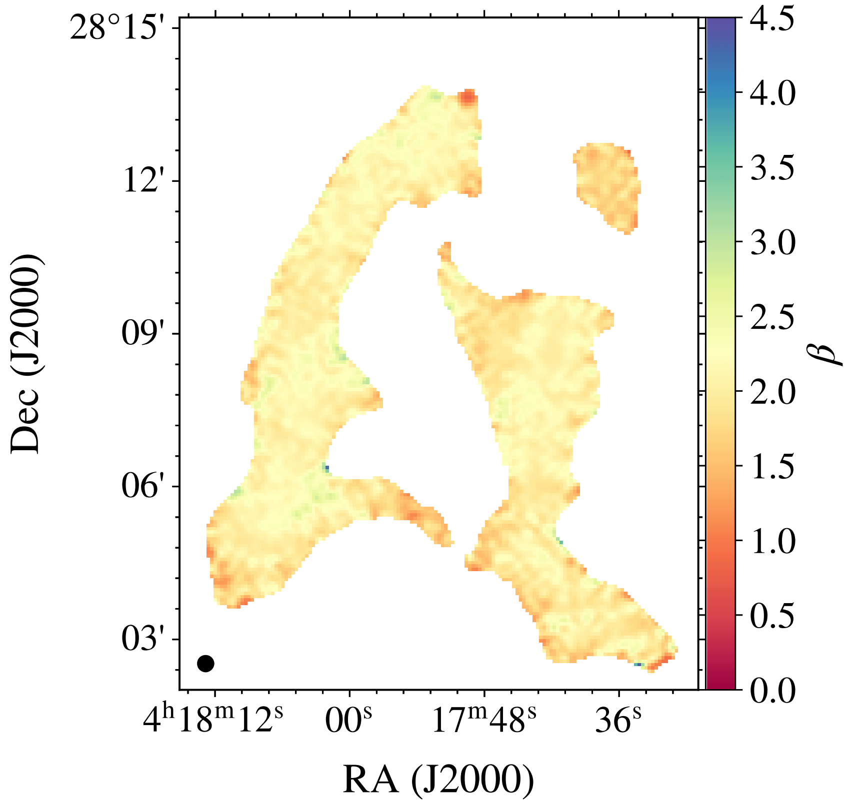
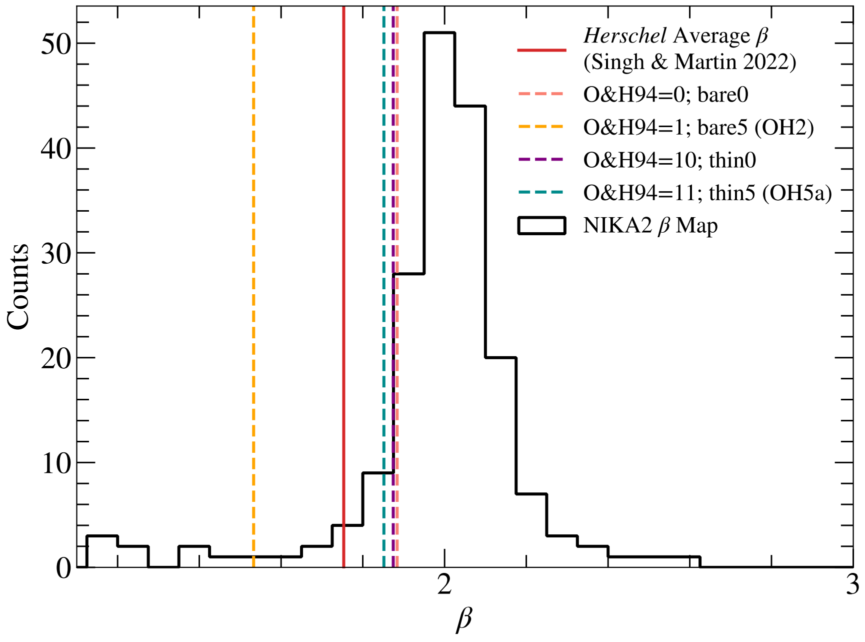


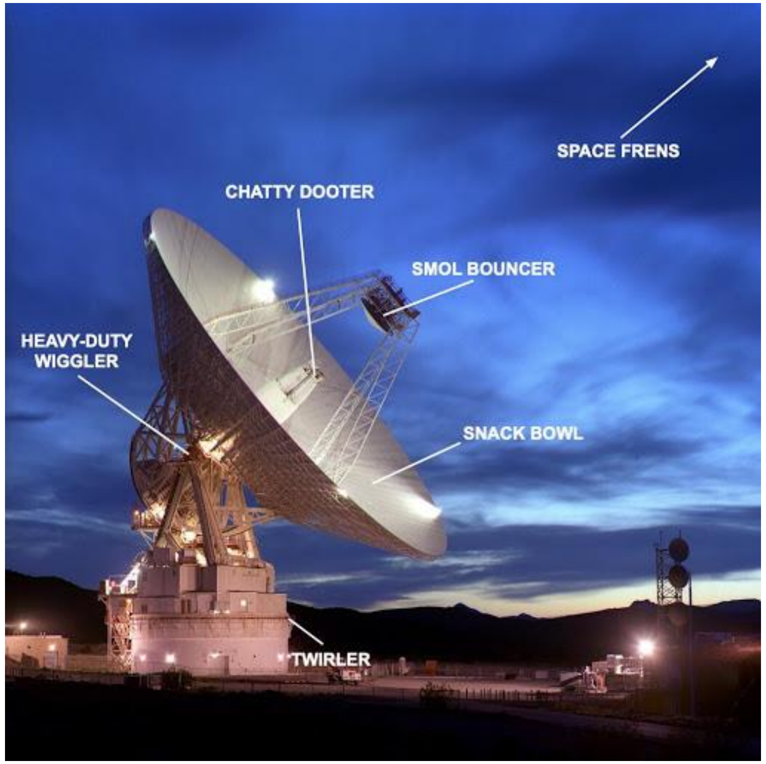


"This suggests that the **opacity laws** we use in our radiative transfer analysis, which are also used elsewhere throughout the literature, **do not accurately describe the observed β**"

Scibelli et al., 2023

It is important to accurately determine dust opacities and grain size distributions! They affect the physical parameters you are interested in determining!! Such as density, temperature, mass, etc.,

Back to... Radio Telescopes and Radiometers
(ERA Chap. 3)



Topic overview:

Sections in **3.1: Antenna Fundamentals**
Sections in **3.2: Reflector Antennas**
Sections in **3.3: Two-Dimensional Aperture Antennas**
Sections in 3.4: Waveguides
Sections in 3.5: Radio Telescopes
Sections in 3.5: Radiometers
Sections in 3.6: Interferometers

Contact: sscibell@nrao.edu

3.1 Antenna Fundamentals

Remember our simple dipole:

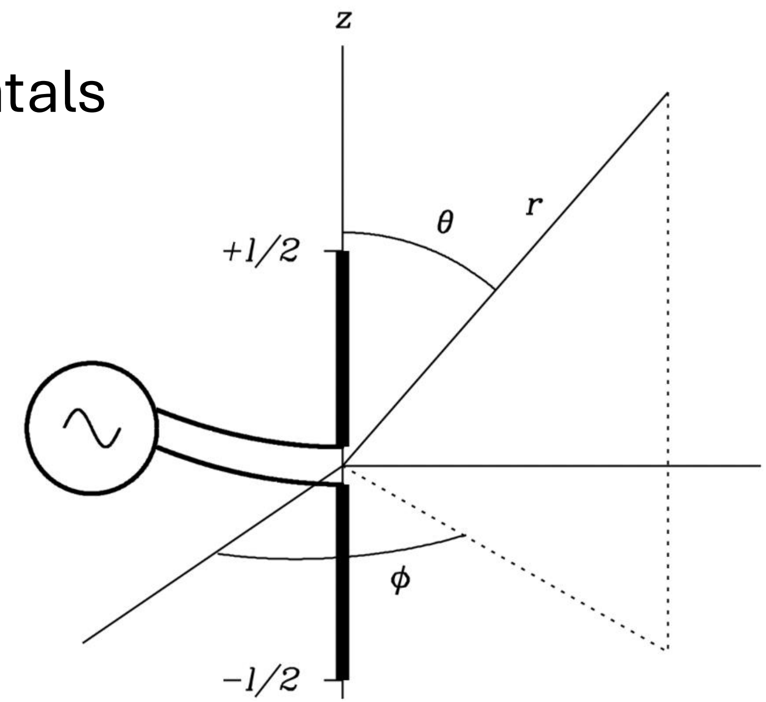


Fig. 3.1

Where the electric current in the wire is defined as the flow rate of the electric charge along the wire:

$$I \equiv \frac{dq}{dt}. \qquad (3.4)$$

On the z-axis, written where v is the instantaneous flow velocity of the charges

$$I = \frac{dq}{dt} = \frac{dq}{dz}\frac{dz}{dt} = \frac{dq}{dz}v, \qquad (3.5)$$

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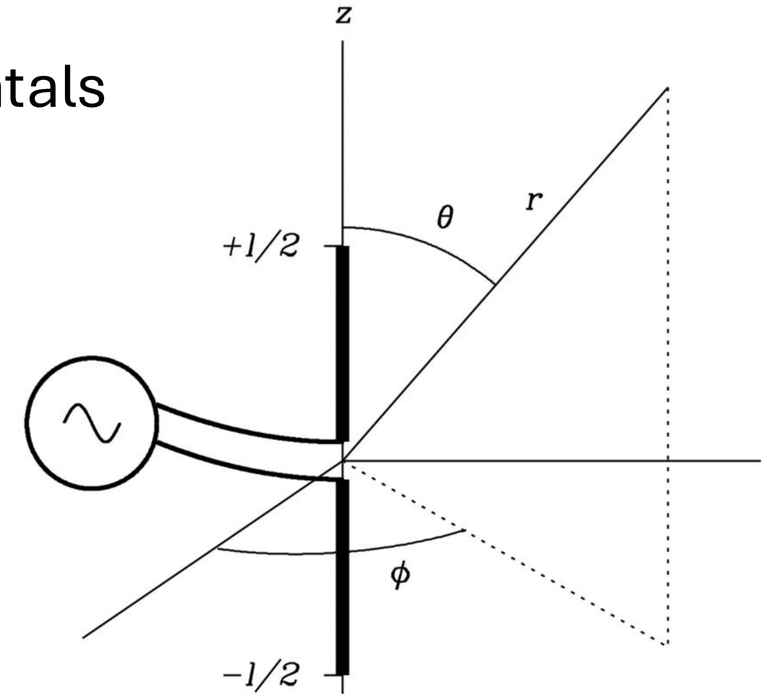

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Electrons together move slowly in a wire, like water out of a garden hose, so Larmor's nonrelativistic equation can be used!
(see book example!)

3.1 Antenna Fundamentals

Remember from Chapter 2! The Power pattern from a 'jerk' of a particle looks like a donut ↓

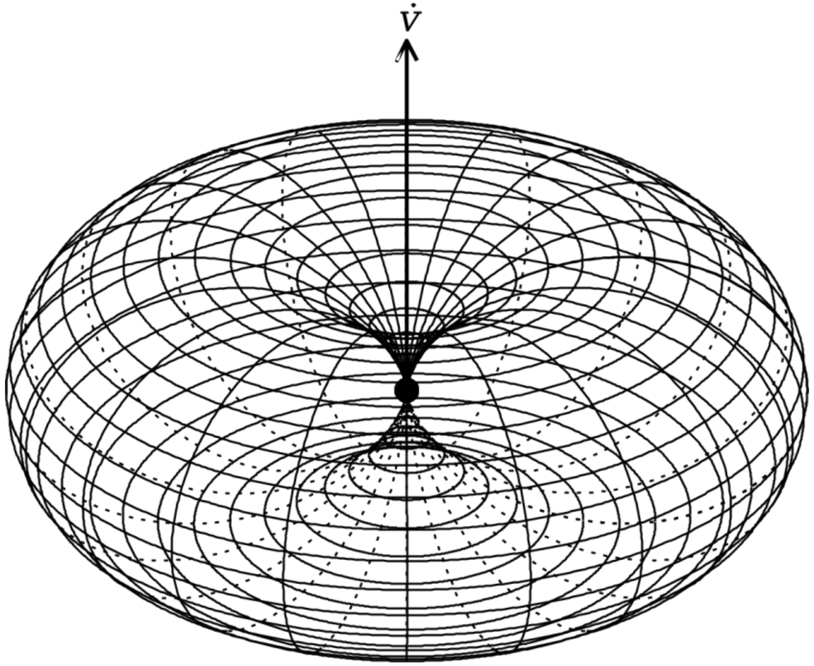

Fig. 2.23

Larmor radiation from a single charged particle

3.1 Antenna Fundamentals

Remember from Chapter 2! The Power pattern from a ‘jerk’ of a particle looks like a donut ↓

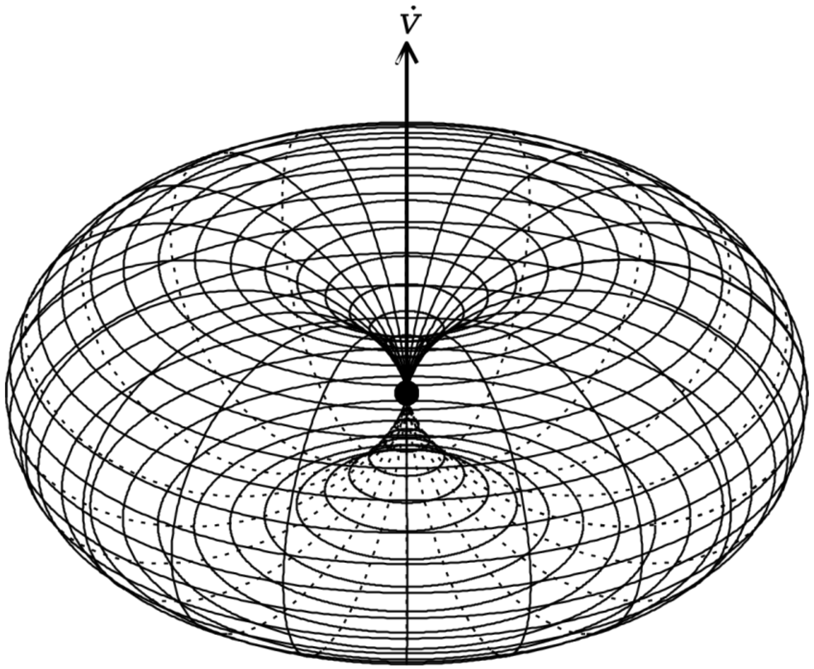

Fig. 2.23
Larmor radiation from a single charged particle

Same **power pattern** we use to describe power from an **antenna**! This time, we are talking about a “charge distribution” where,

$$P \propto \sin^2 \theta. \quad (3.14)$$

And the **time-averaged total power** emitted is obtained by integrating the Poynting flux over the surface area of a sphere of any radius $r \gg l$ centered on the antenna

$$\langle P \rangle = \frac{\pi^2}{3c}\left(\frac{I_0 l}{\lambda}\right)^2, \quad (3.17)$$

3.1 Antenna Fundamentals

Half-wave dipoles

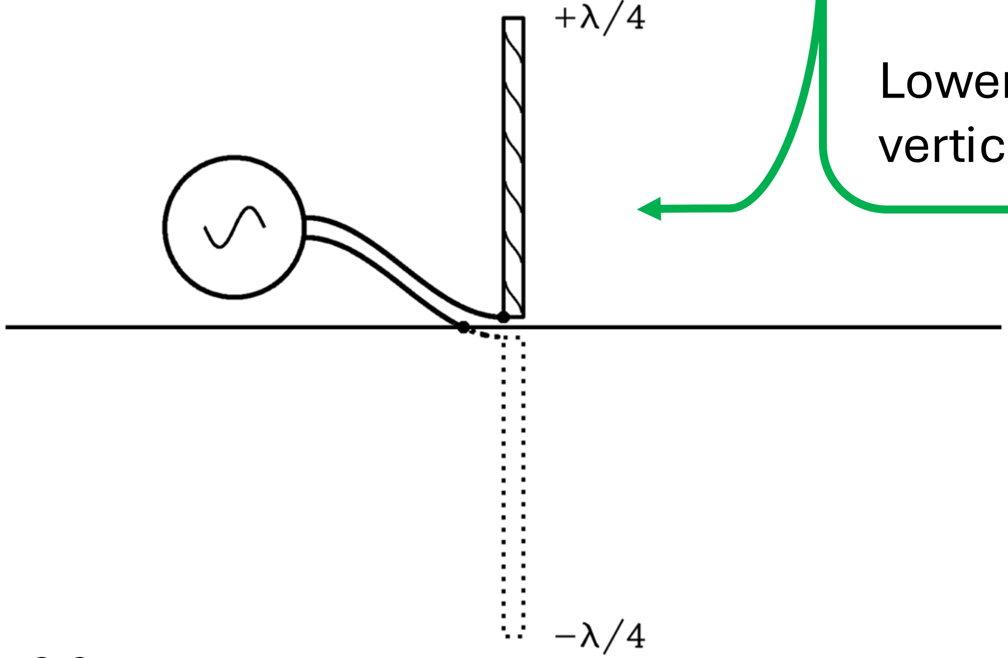

Fig. 3.2
A ground-plane vertical antenna is just half of a dipole above a conducting plane

Most practical dipoles are half-wave dipoles or **quarter-wave ground-plane verticals**

Lower half of dipole is the reflection of the vertical - same process as a mirror