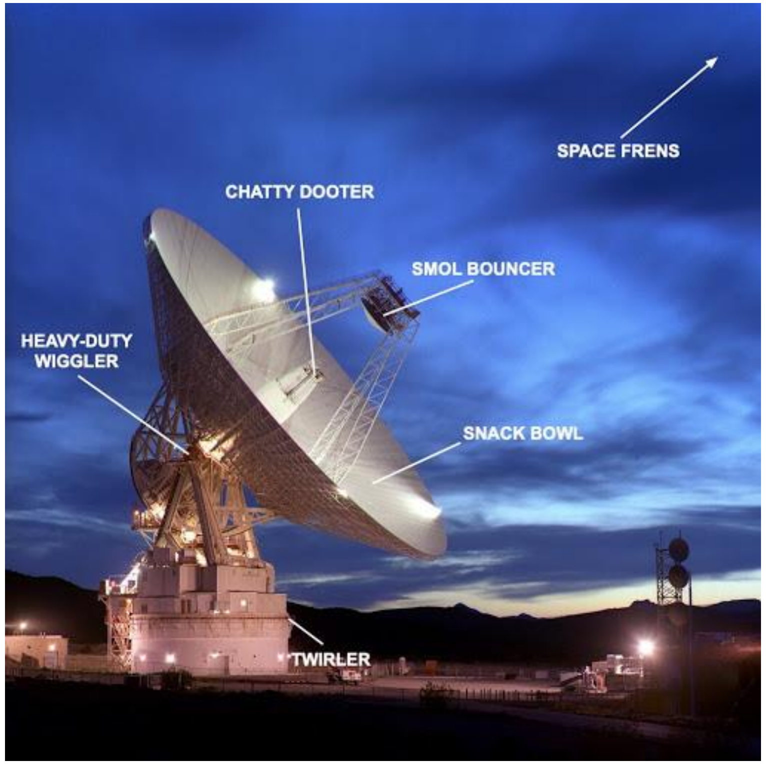
Radio Telescopes and Radiometers

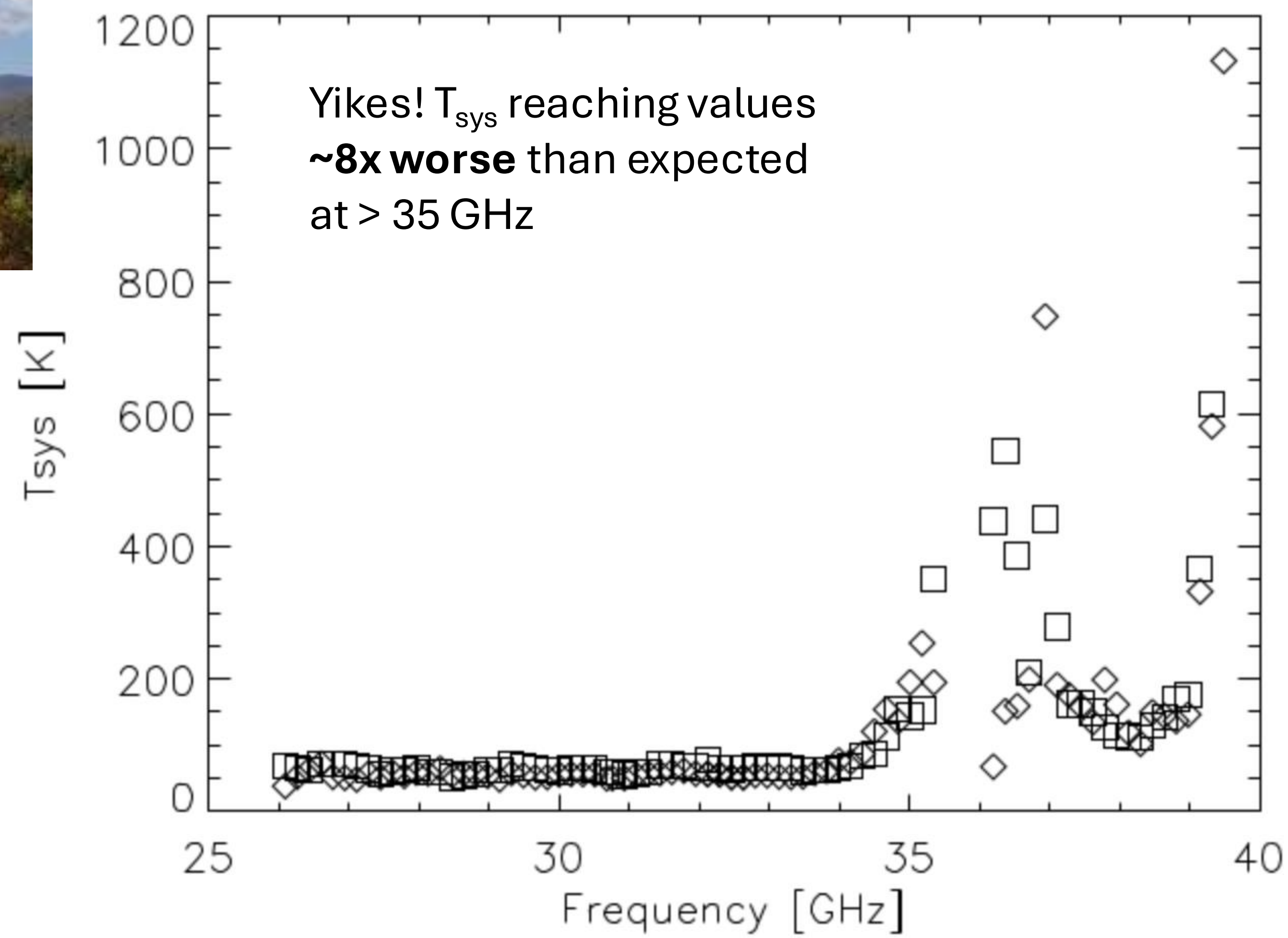
(ERA Chap. 3)



Topic overview:

Sections in 3.1: Antenna Fundamentals
Sections in 3.2: Reflector Antennas
Sections in 3.3: Two-Dimensional Aperture Antennas
Sections in 3.4: Waveguides
Sections in 3.5: Radio Telescopes
Sections in 3.6: Radiometers
Sections in 3.7: Interferometers

Contact: sscibell@nrao.edu
GREEN BANK OBSERVATORY
National Radio Astronomy Observatory



Question: How much <u>longer</u> would it take to integrate to reach the same rms level ?

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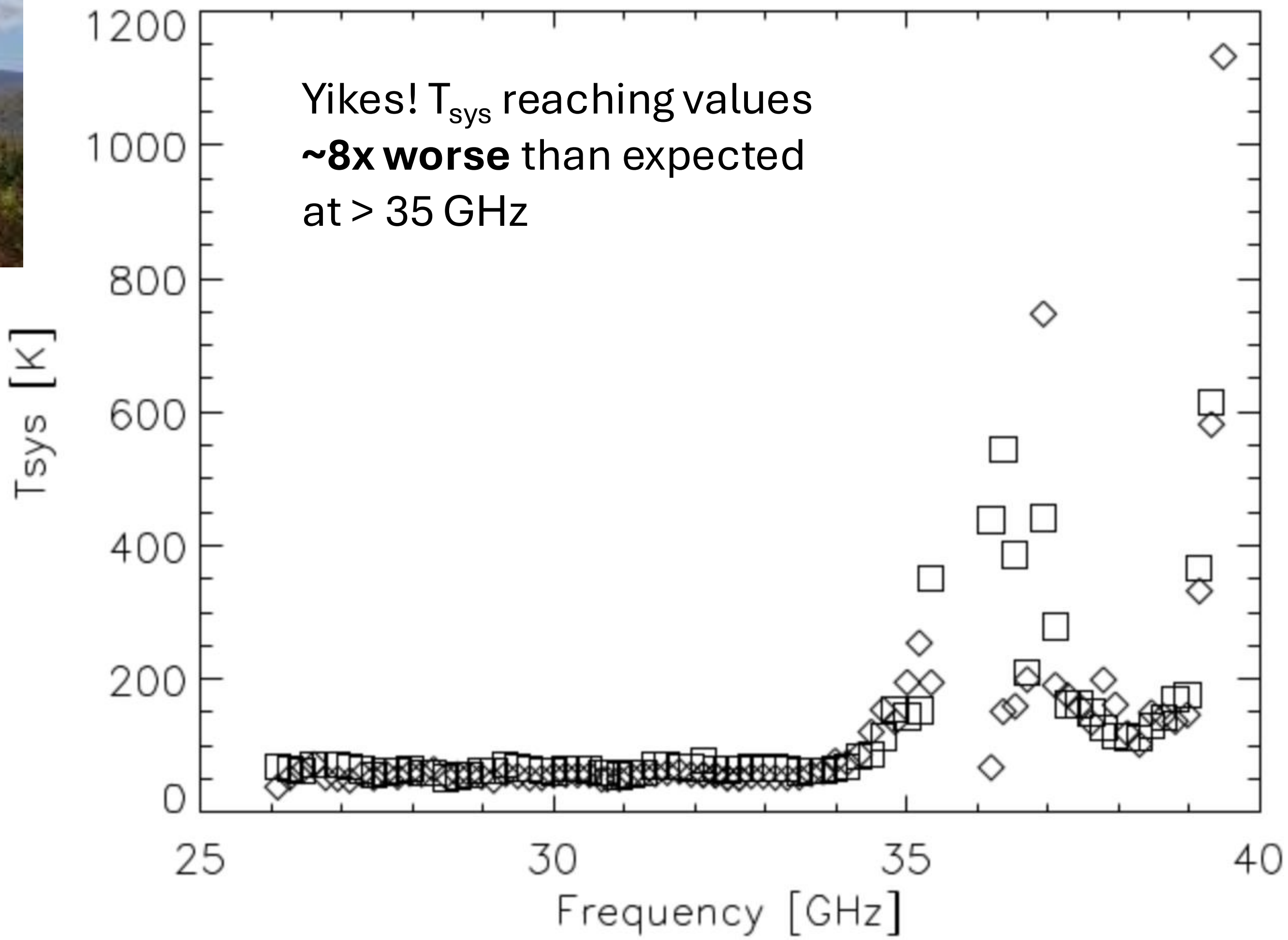


Yikes! T_{sys} reaching values **~8x worse** than expected at > 35 GHz

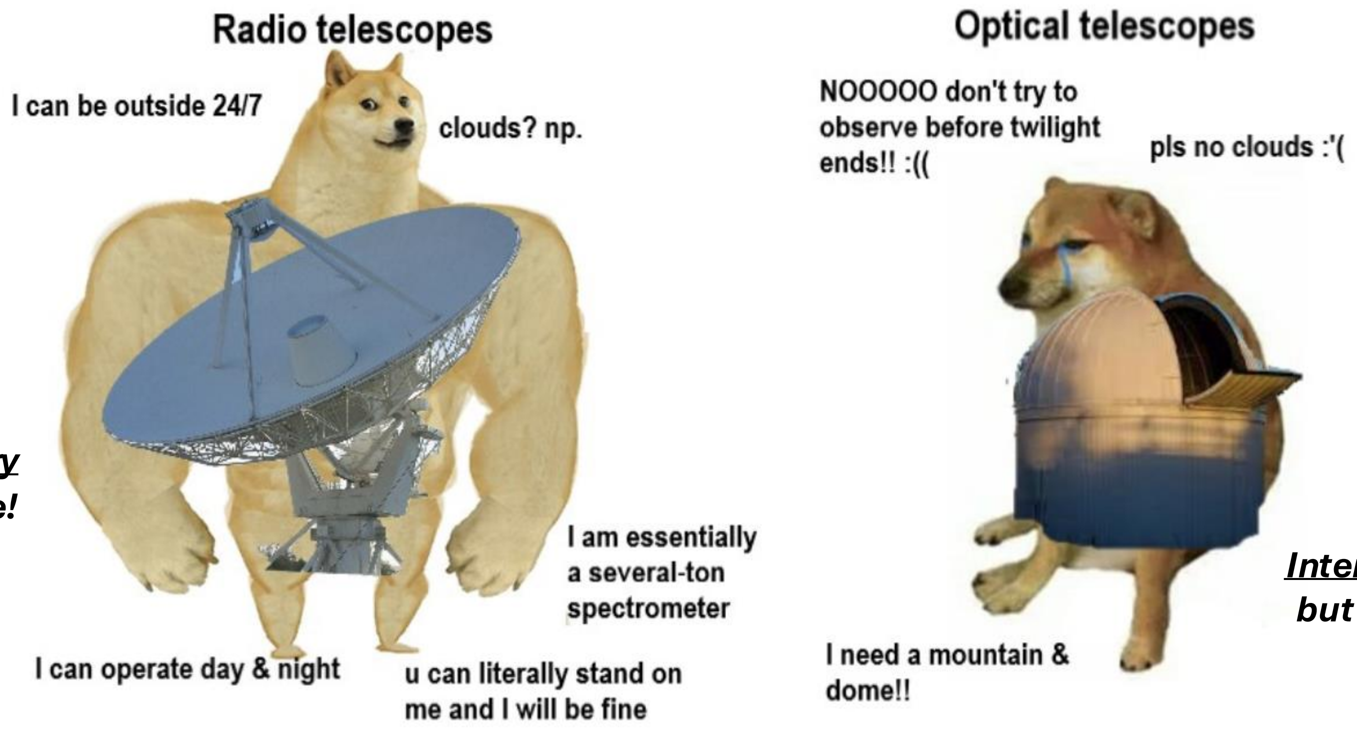
Question: How much <u>longer</u> would it take to integrate to reach the same rms level ?

64x longer!

$$\sigma_T = \frac{2T_s}{\sqrt{\Delta\nu\,\tau}}.$$

Interferometers and Interferometry

(ERA Chap. 3.7, THz Astronomy Chap. 9)



***Interferometry* is easy for me!**

I can do Interferometry… but its harder :(

Introduction – Why Interferometers?

Reminder: The size of the smallest angular structure that can be resolved by a **single dish telescope** operating in the diffraction **limit** is given by,

$$\theta \sim \lambda/D \text{ radians}$$

Disadvantages of a single dish:

- Usually one (or only several) 'pixel(s)' or beam(s) with limited resolution
- Hard to make one dish bigger and bigger...
- Due to gravitational sagging, telescope deformations, torques (b/c wind) this causes **limits in the mechanical tracking and pointing accuracies** (best possible ~ 1 arcsec)

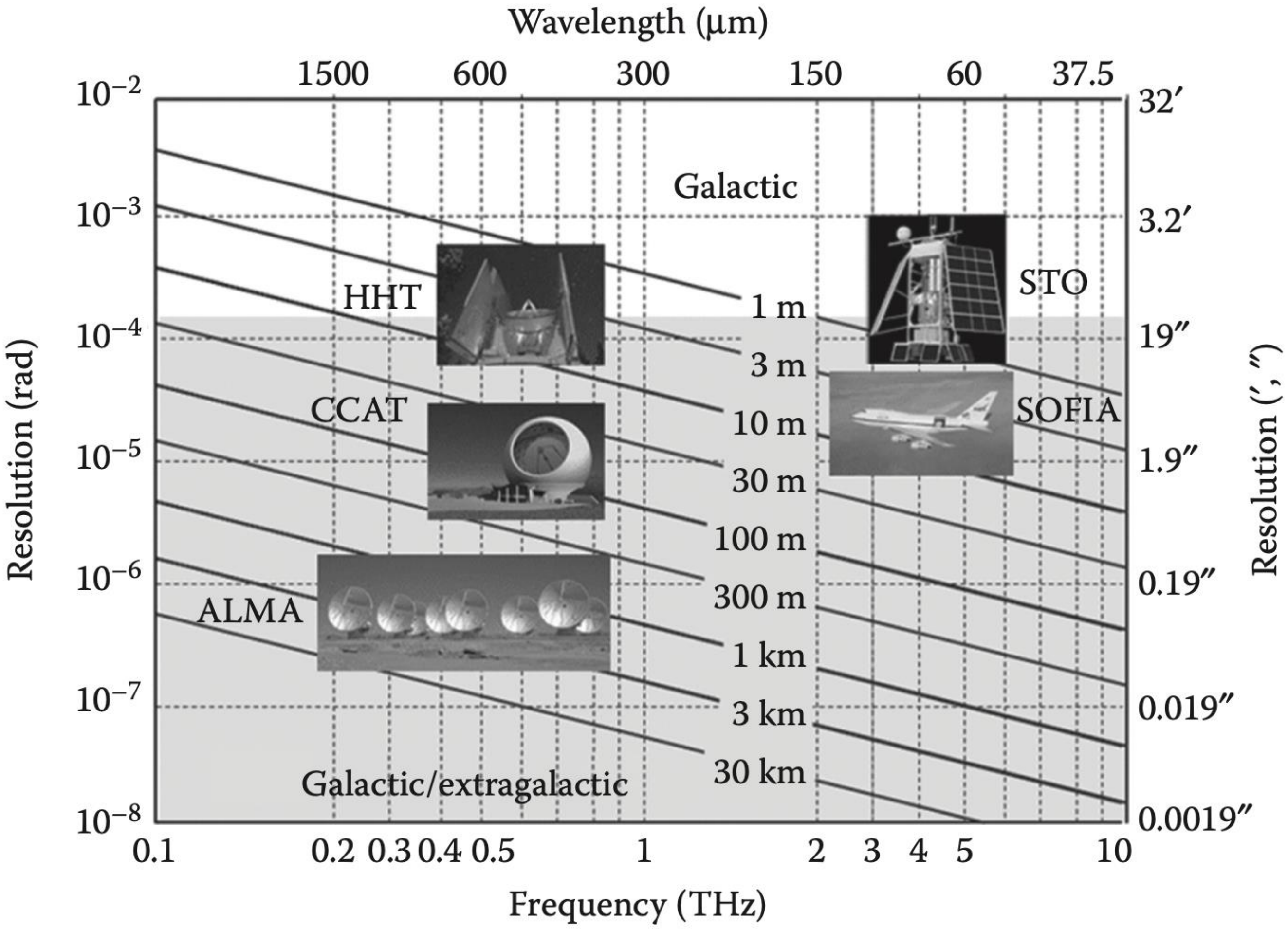


Fig. 9.1 (THz Astronomy)

Introduction – Why Interferometers?

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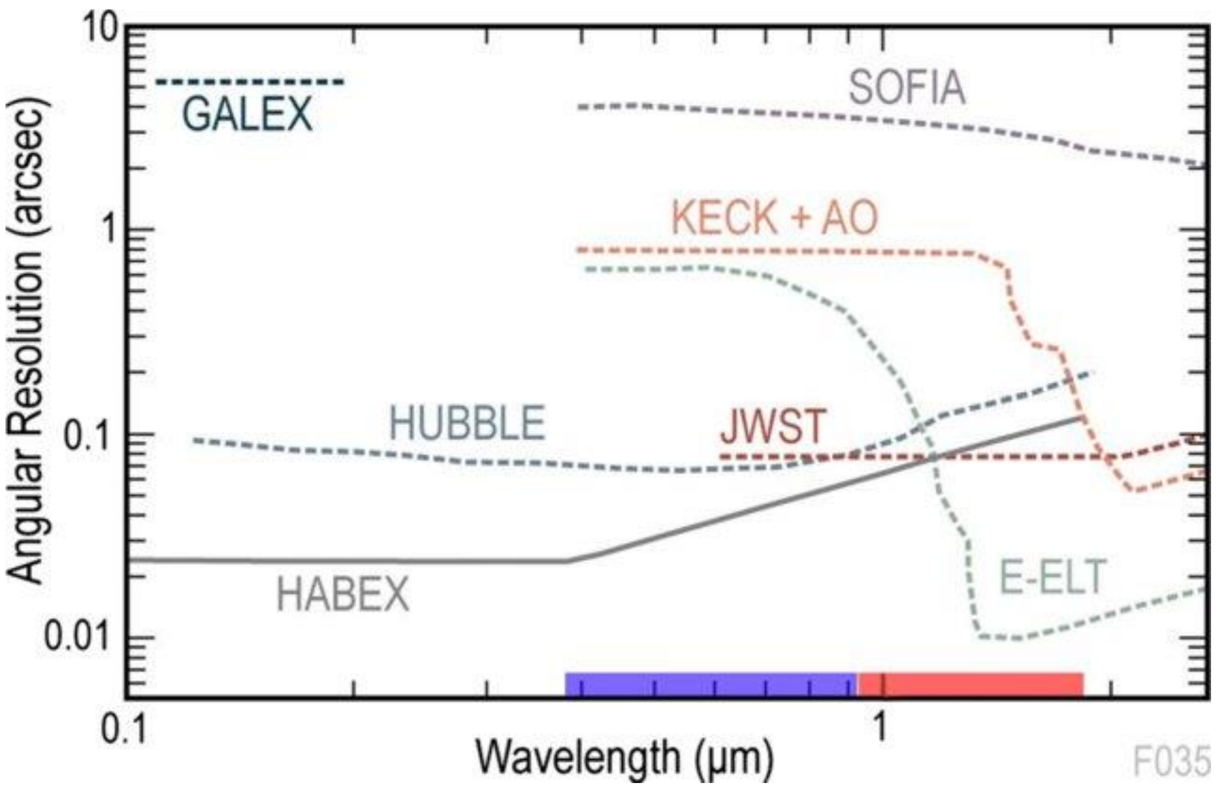
θ ~ λ/D radians

Question: What <u>size</u> telescope would we need to match Hubble or JWST angular resolution for 1cm observations?

Most UV/Visible/IR telescopes make high resolution images



Jahnke et al., 2021

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Question: What <u>size</u> telescope would we need to match Hubble or JWST angular resolution for 1cm observations? ~25km!

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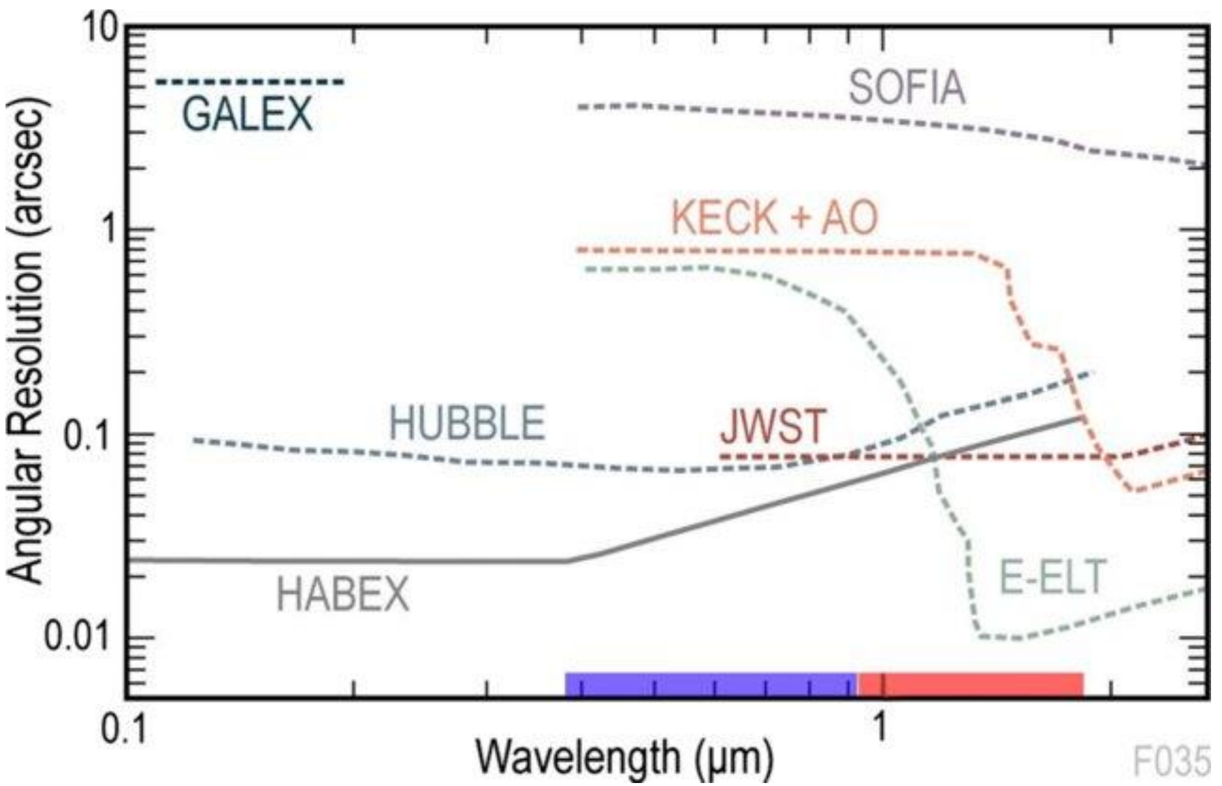

Jahnke et al., 2021

Interferometer Definition:

An interferometer combines the **light collected by two or more telescopes** to **create a power pattern** sensitive to **angular size scales proportional to the projected separation** (i.e., baselines) between them.

Angular structure that can be resolved by an **interferometer still** operates in the diffraction **limit,** however now:

$\theta \sim \lambda/b$ radians

Where ‘b’ is the projected baseline separation

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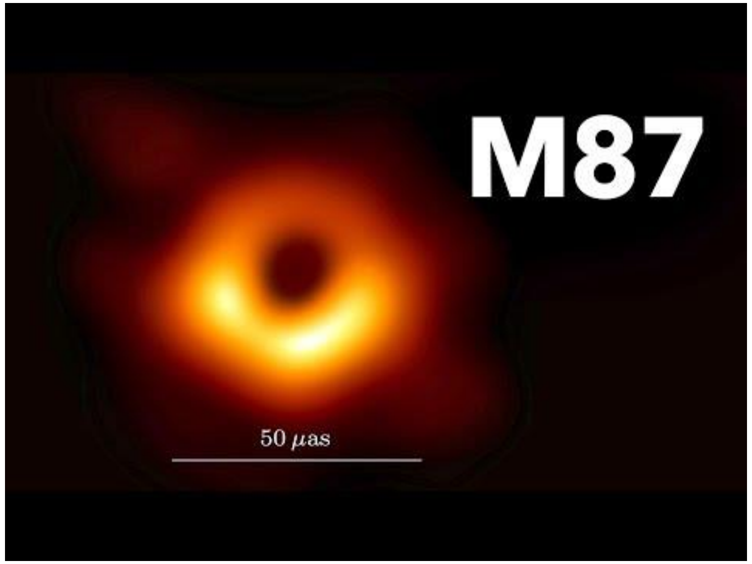

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- Each antenna/parabola has its own coherent detection system that preserves knowledge of the amplitude and phase of the arriving photons

- This knowledge can be used in **real time** or or later (**even years later**!) to allow the photons to interfere yielding the telescope's beam at the time of the observation

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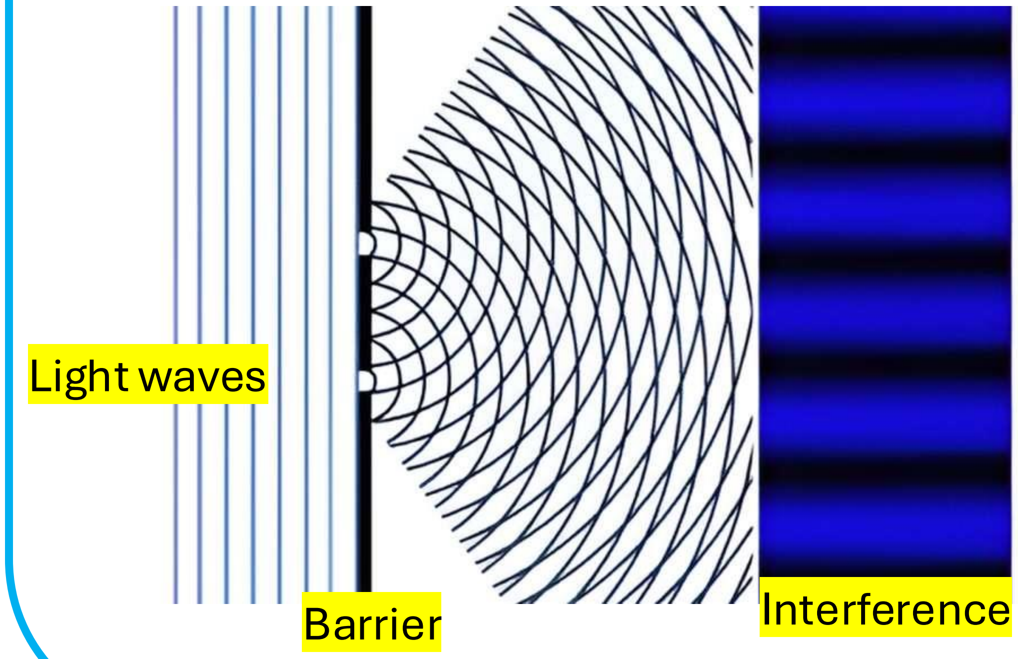
IN A NUTSHELL:
Interferometers synthesize a large telescope from a number of smaller ones by preserving knowledge of the arriving photons' amplitude and phase

1st A Look at Historical 'Adding' Interferometers

It's all about the 'fringe'!

The law of interference of light was described in the **Young's double slit experiment** that demonstrated interference fringes:



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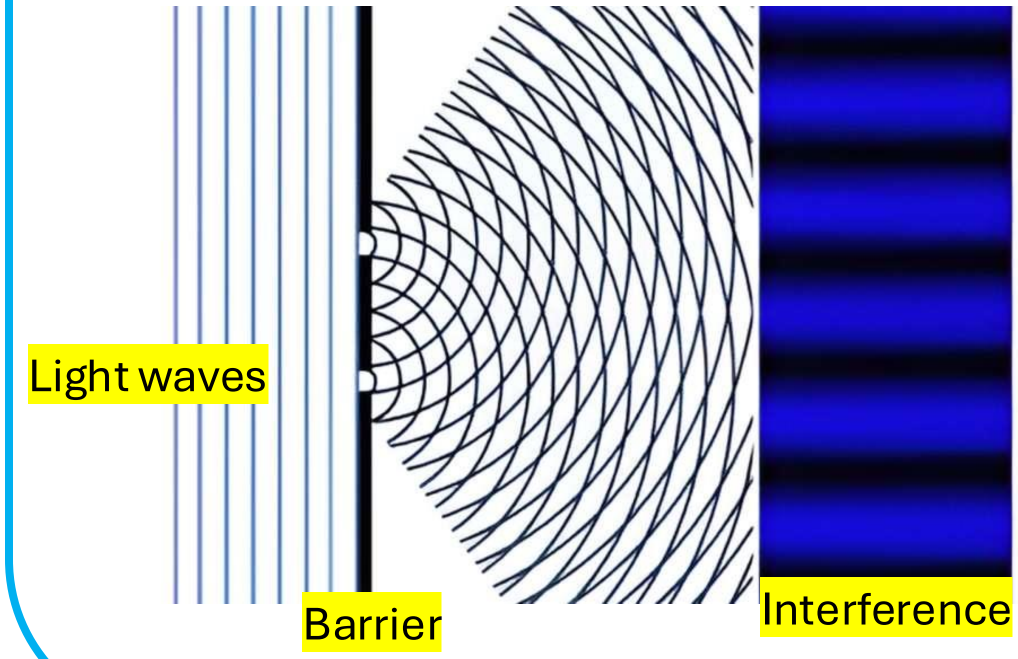
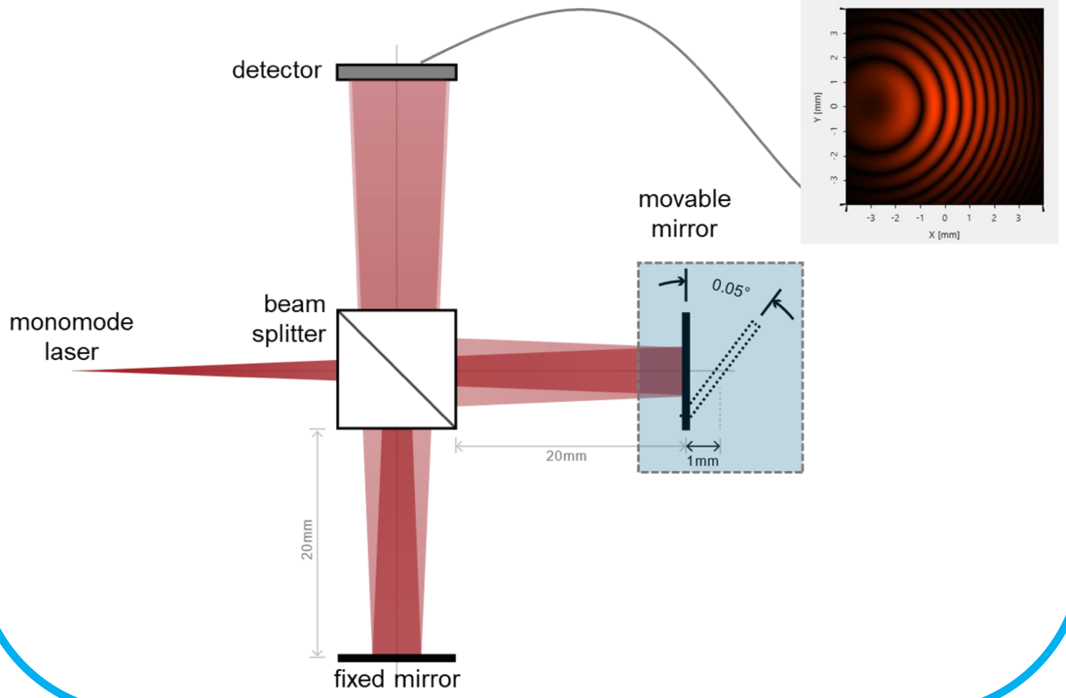


To extract out information from this interference use an interferometer!
E.g., a **Michelson interferometer**



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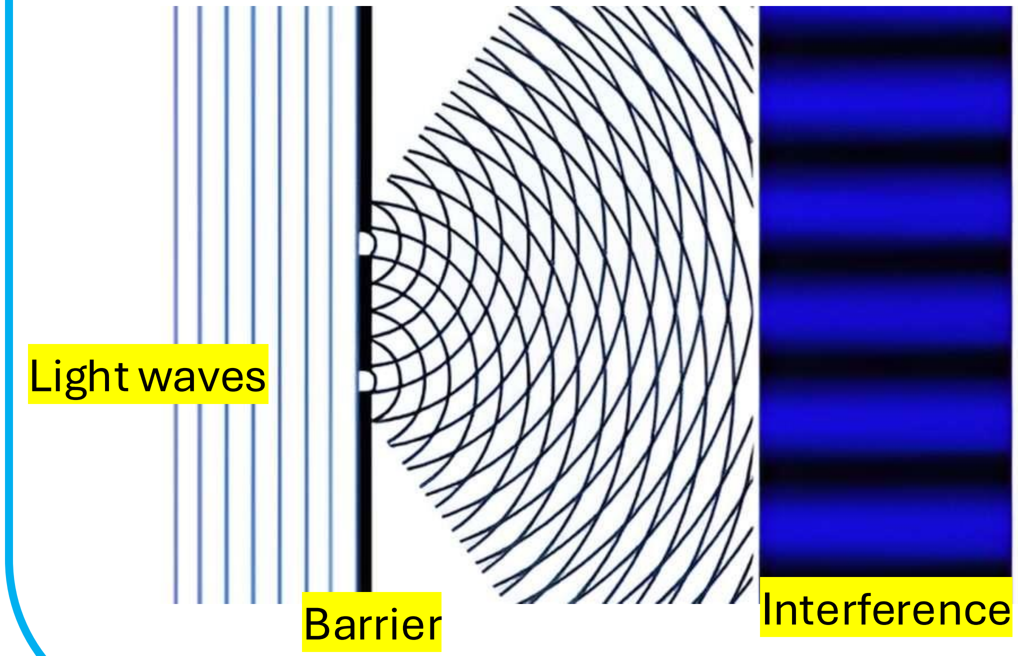

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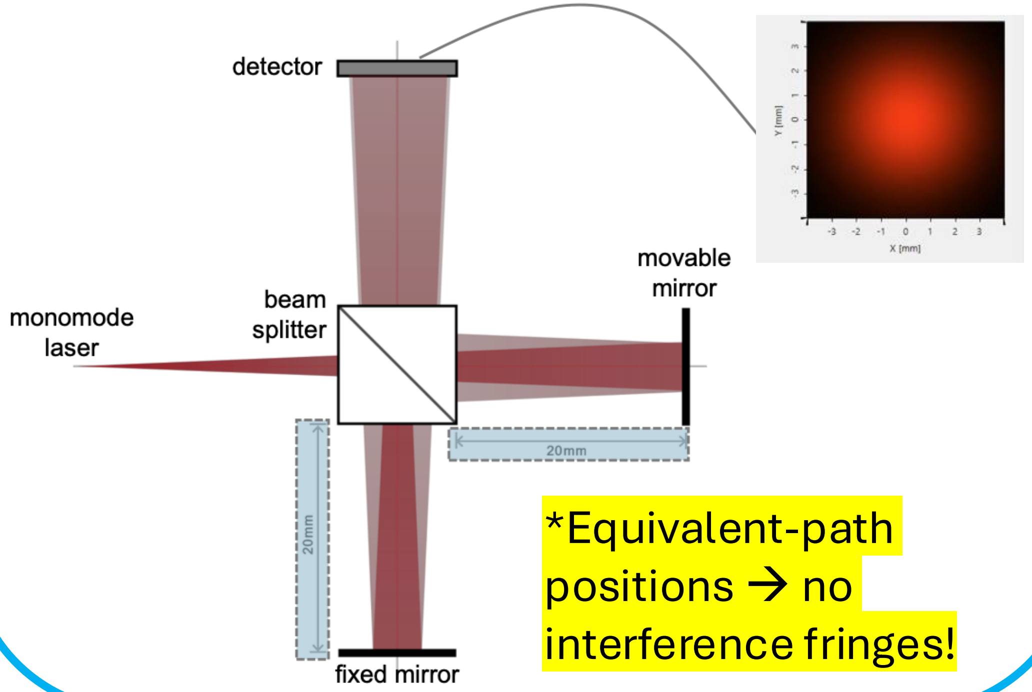

*Equivalent-path positions → no interference fringes!

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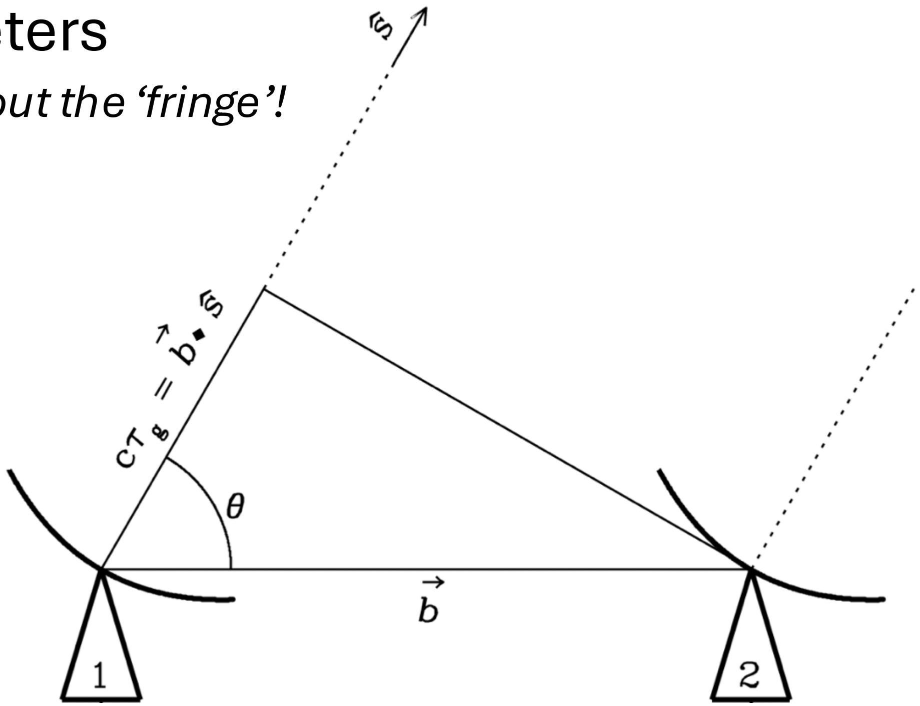
It's all about the 'fringe'!

Fig. 3.41 (ERA)



The simple two-element radio interferometer is set up so they are separated by baseline vector of length *b* that points from antenna 1 to antenna 2

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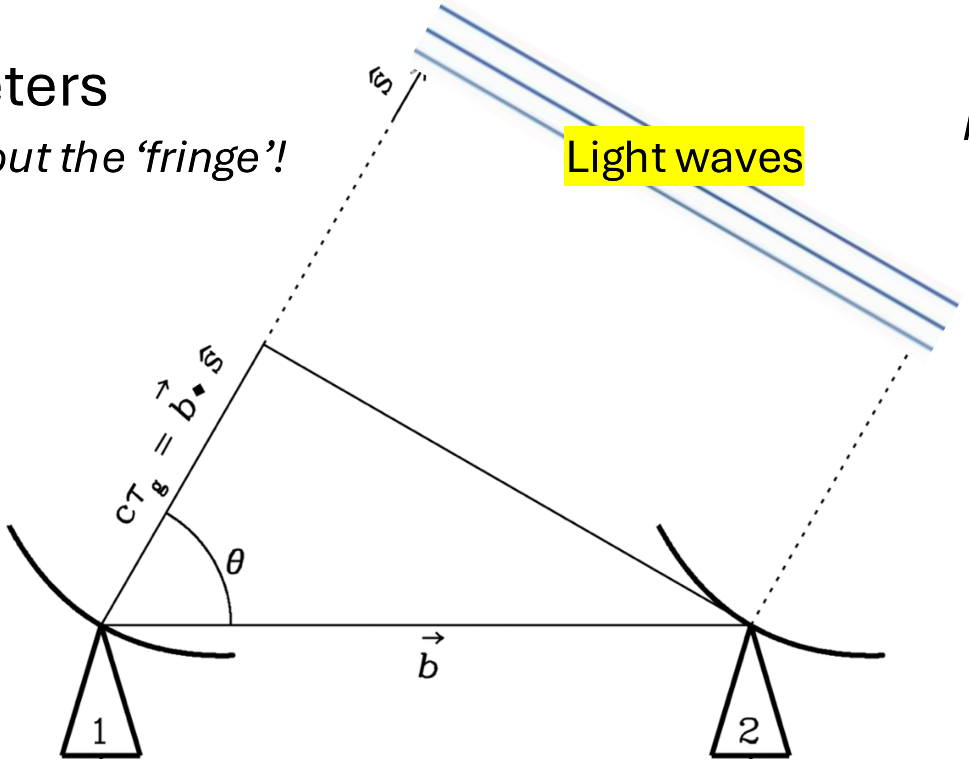

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Plane waves must travel an extra distance to reach antenna 1, $\vec{b} \cdot \hat{s} = b \cos\theta$, so there is a **geometric time delay**:

$$\tau_g = \frac{\vec{b} \cdot \hat{s}}{c}. \qquad (3.172)$$

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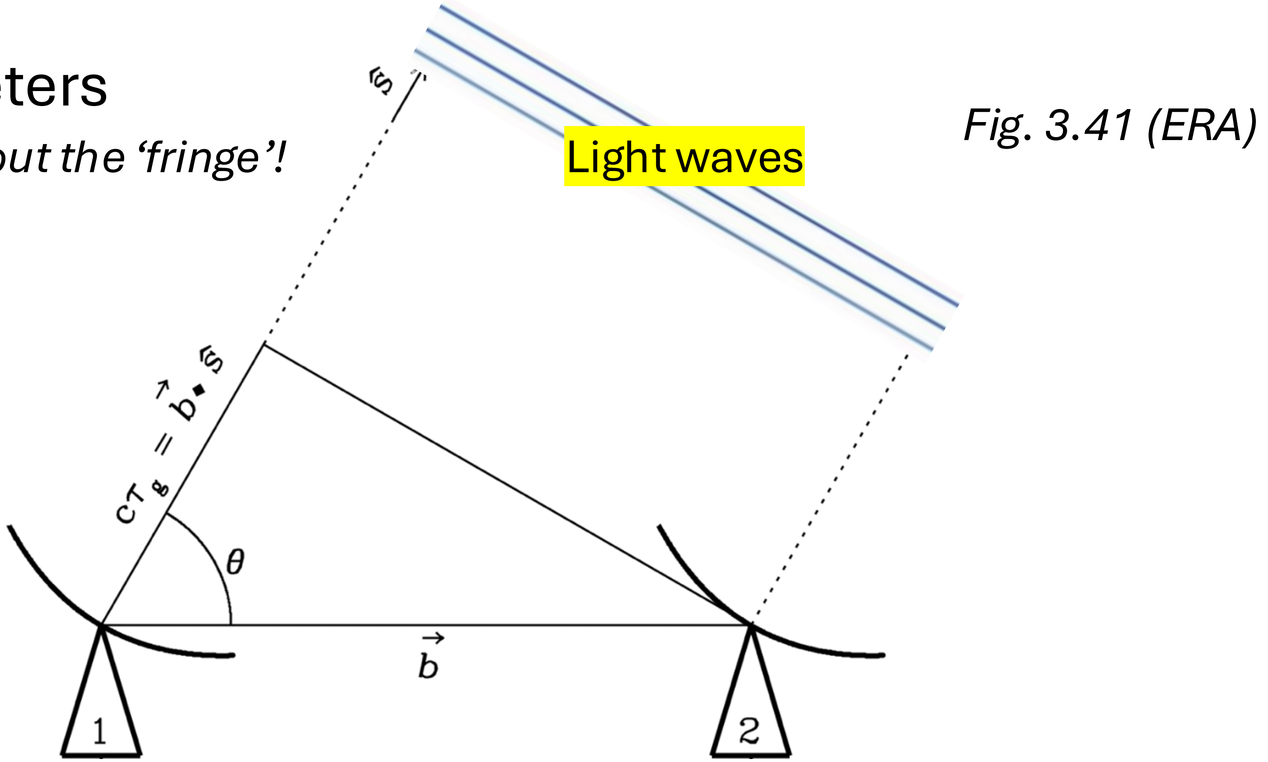

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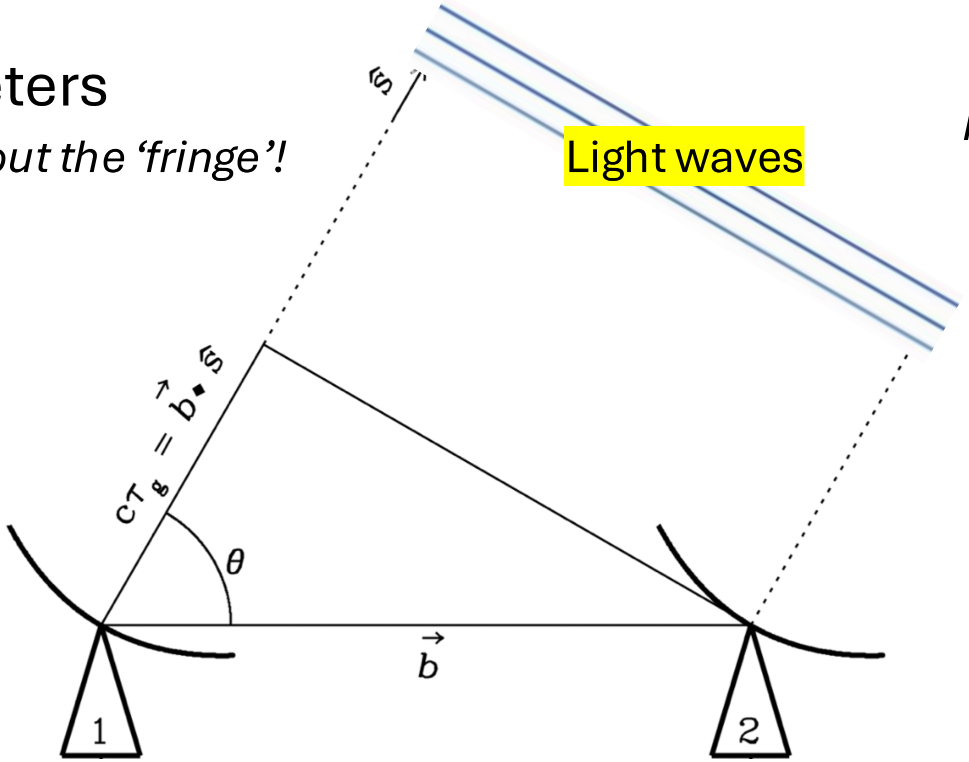
- This delay causes **fringes** directly related to the projected baseline!
- Dependence on time allows for very precise measurements of position at high accuracy

1^{st} A Look at Historical 'Adding' Interferometers

It's all about the 'fringe'!



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We write for a fringe with angular shift,

$$\Delta\theta = \lambda / (b \sin\theta)$$

*This defines **only one Fourier component** of the sky brightness – need more b's !

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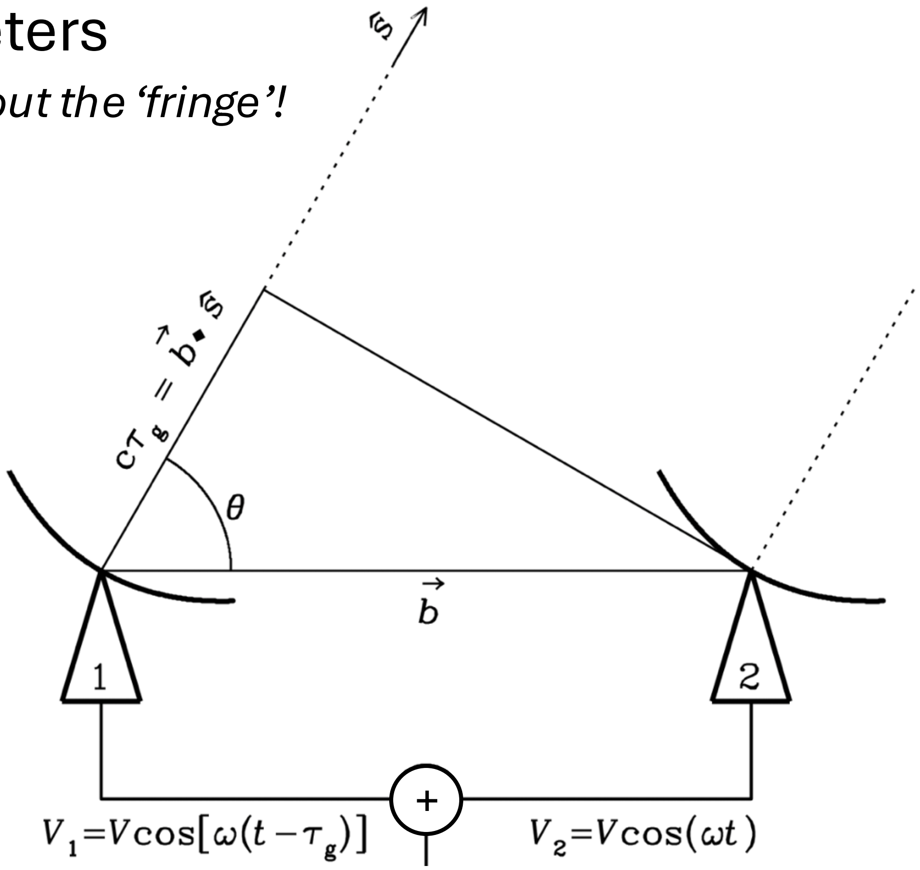
1^{st} A Look at Historical 'Adding' Interferometers

It's all about the 'fringe'!

Fig. 3.41 (ERA)

The first interferometers were essentially radio versions of the Michelson interferometer (Ryle & Vonberg, 1946)

- Combines signals at the same frequency
- The uncorrelated noise power of the two antennas are simply added together
- The projected baseline (b) changes as the source first rises and then sets



Output is $\propto (V1 + V2)^2$

$\propto V1^2 + 2V1V2 + V2^2$

1st A Look at Historical 'Adding' Interferometers

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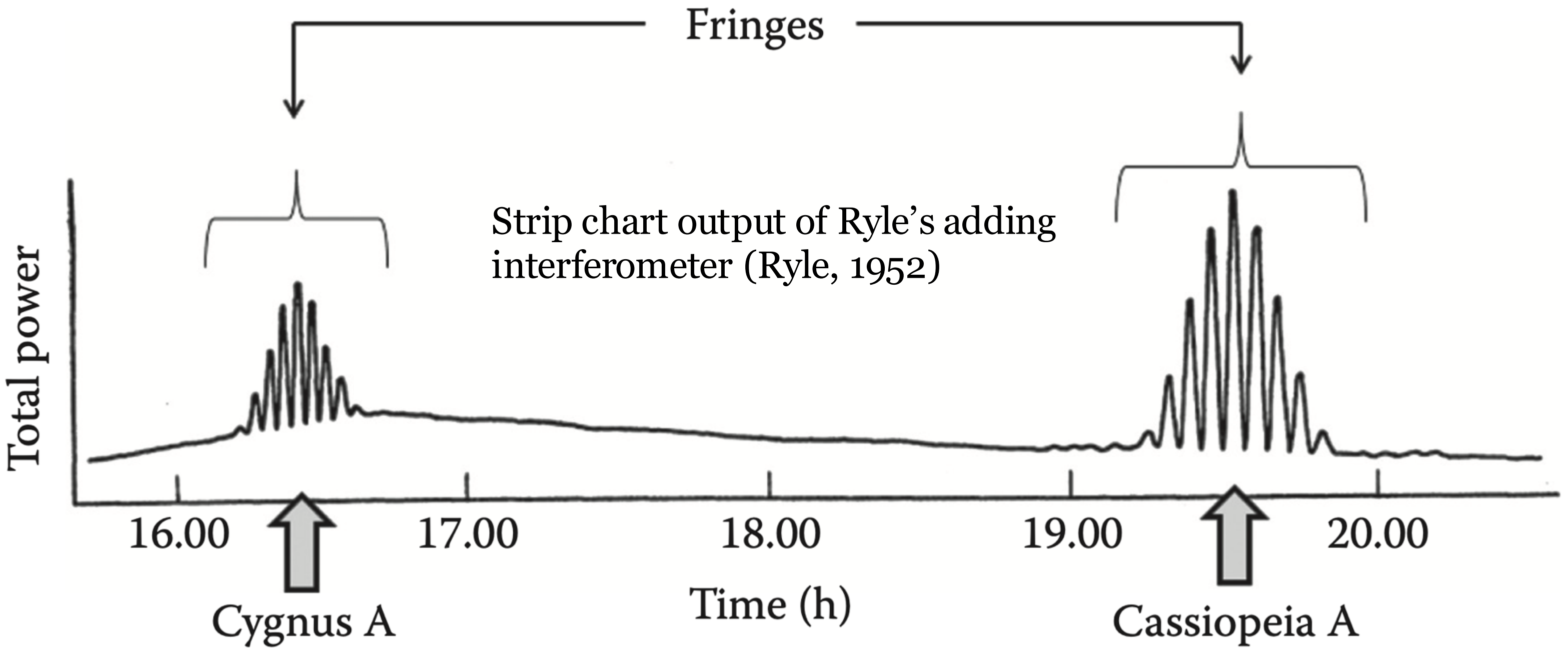

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1st A Look at Historical 'Adding' Interferometers

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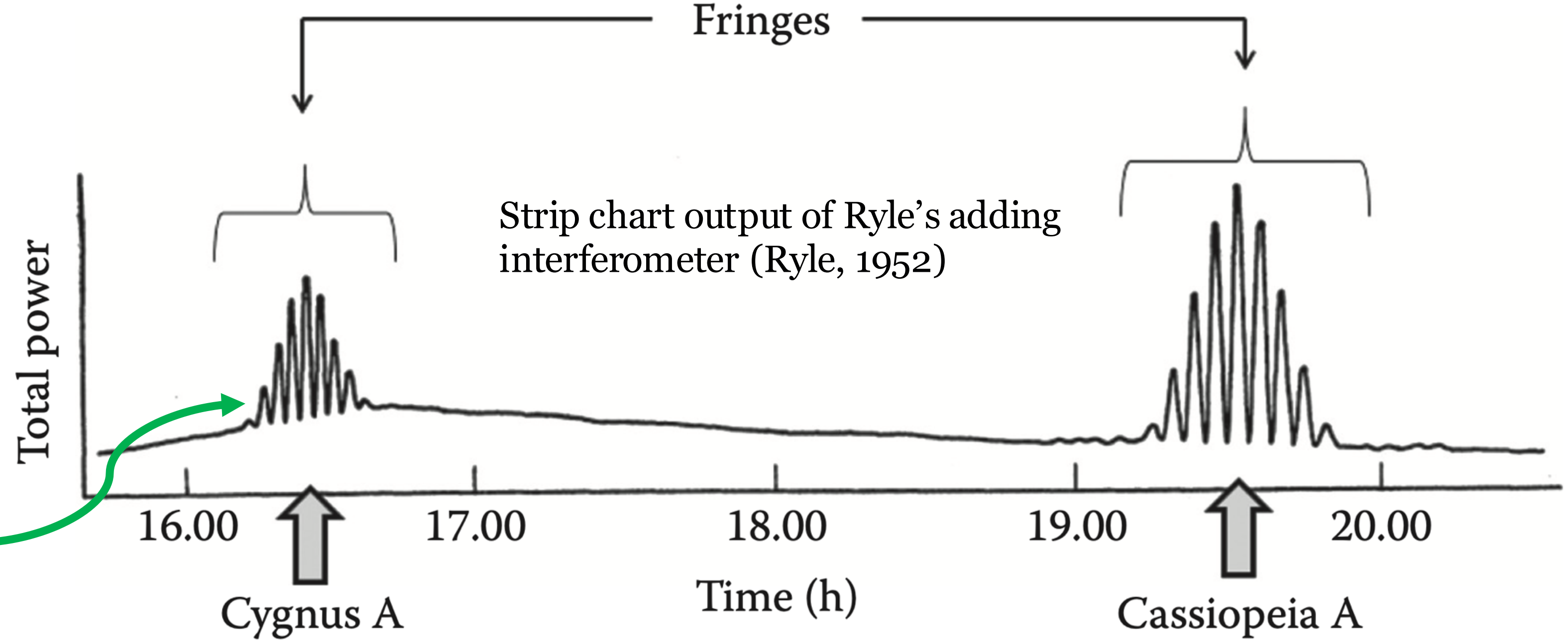
Fig. 9.3 (THz Astronomy)

The null depths in the observed fringes go all the way to the noise floor, indicating that the **angular size of the astronomical source that passed through them was smaller in extent than the fringes themselves** – AKA the source is unresolved



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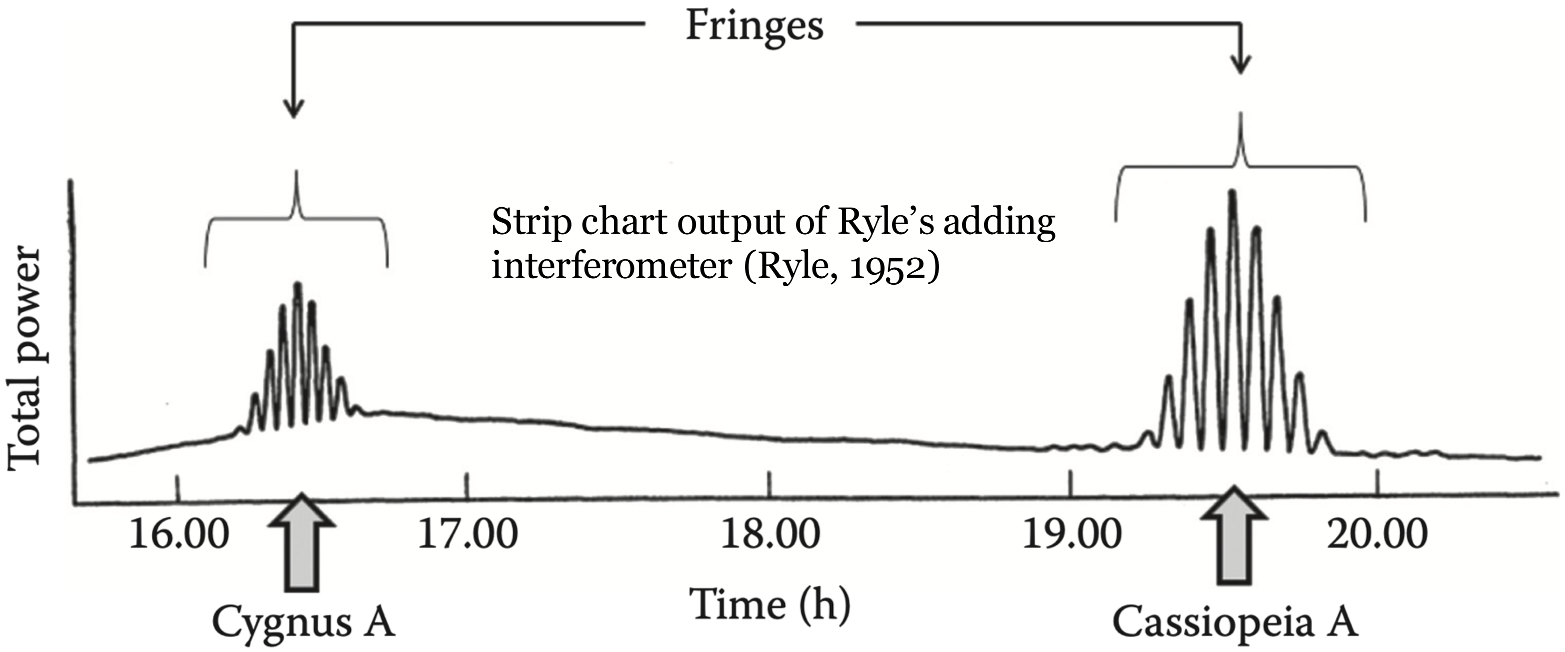
1^{st} A Look at Historical 'Adding' Interferometers

It's all about the 'fringe'!

Fig. 9.3 (THz Astronomy)

Most sources will have both small and extended spatial extents

If **no fringes** are observed, the characteristic angular frequencies associated with the source are much larger than the fringe width and the source is said to be **"resolved-out"** by the interferometer



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1st A Look at Historical 'Adding' Interferometers

It's all about the 'fringe'!

As a way of quantifying the relative amplitude of fringes a **fringe visibility**, V_M, is defined (Thompson et al., 1991):

$$V_M = \frac{\text{brightness of maxima} - \text{brightness of minima}}{\text{brightness of maxima} + \text{brightness of minima}} = \frac{S_{max} - S_{min}}{S_{max} + S_{min}}$$ (9.8 *in THz Astronomy*)

Where,

S_{max} = maximum observed source flux (Jy)
S_{min} = minimum observed source flux (jy)

And therefore,

$$V_M = 1 \Rightarrow \text{source unresolved}$$
$$1 > V_M > 0 \Rightarrow \text{source partially resolved}$$
$$V_M = 0 \Rightarrow \text{source resolved-out}$$

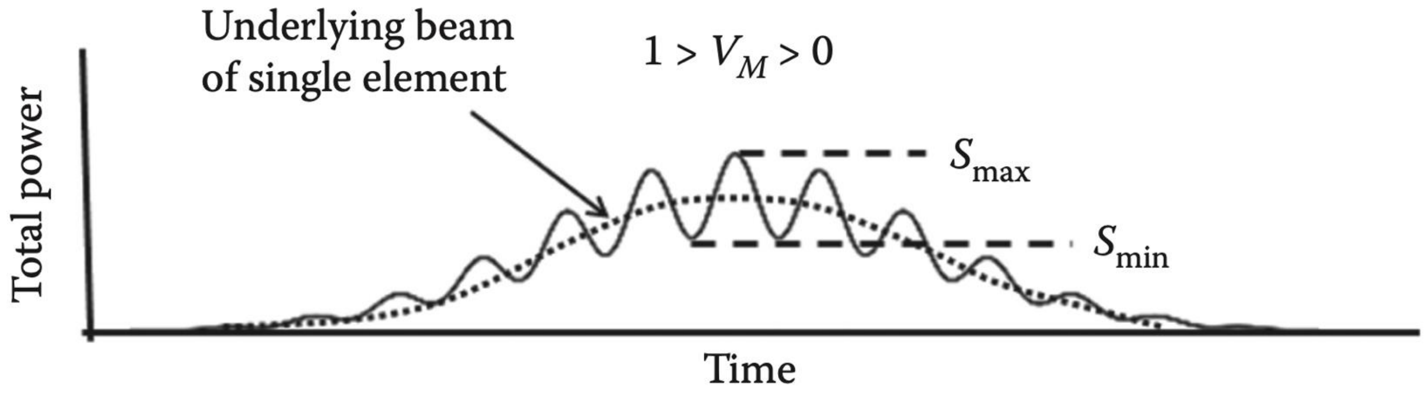


Fig. 9.4 (THz Astronomy)

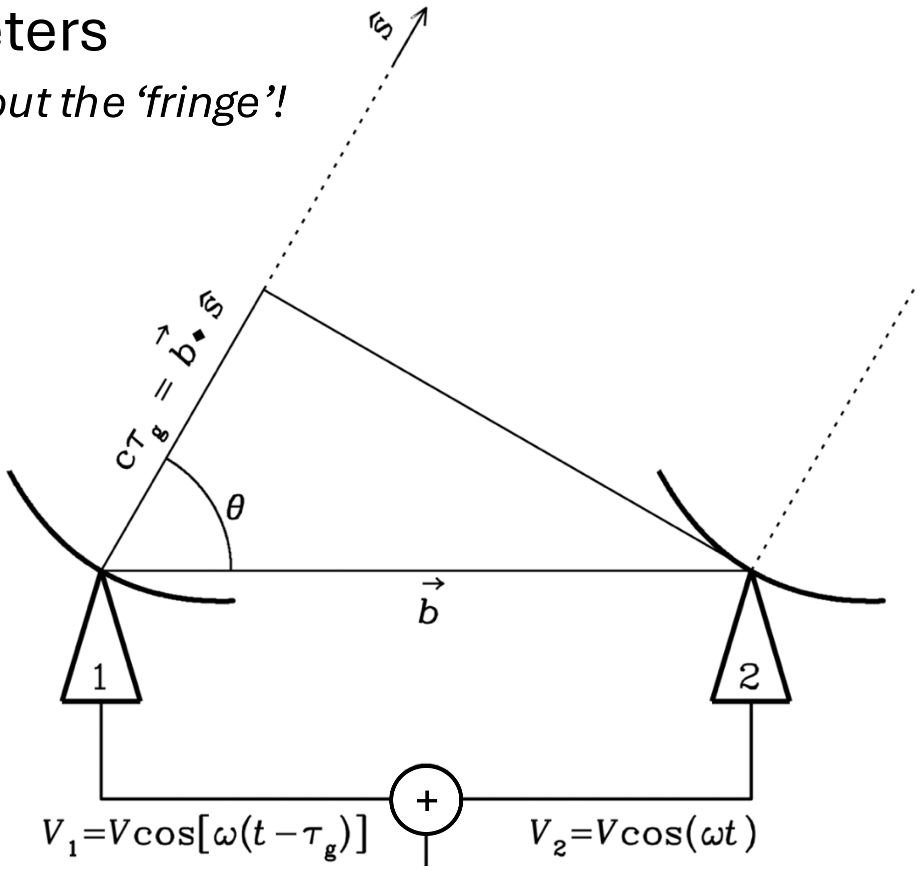
1^{st} A Look at Historical 'Adding' Interferometers

It's all about the 'fringe'!

Adding interferometers:

- These are still useful in incoherent systems, such as optical interferometers
- ISSUES:
    - The signal is combined in a coaxial cable *before reaching the receiver*
    - Thus, they are limited by **needing to have antennas close together**, in order to limit signal loss in the transmission lines

Let's look at modern day 'multiplying' interferometers...



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Multiplying or Coherent Interferometers:

It's still all about the 'fringe'!

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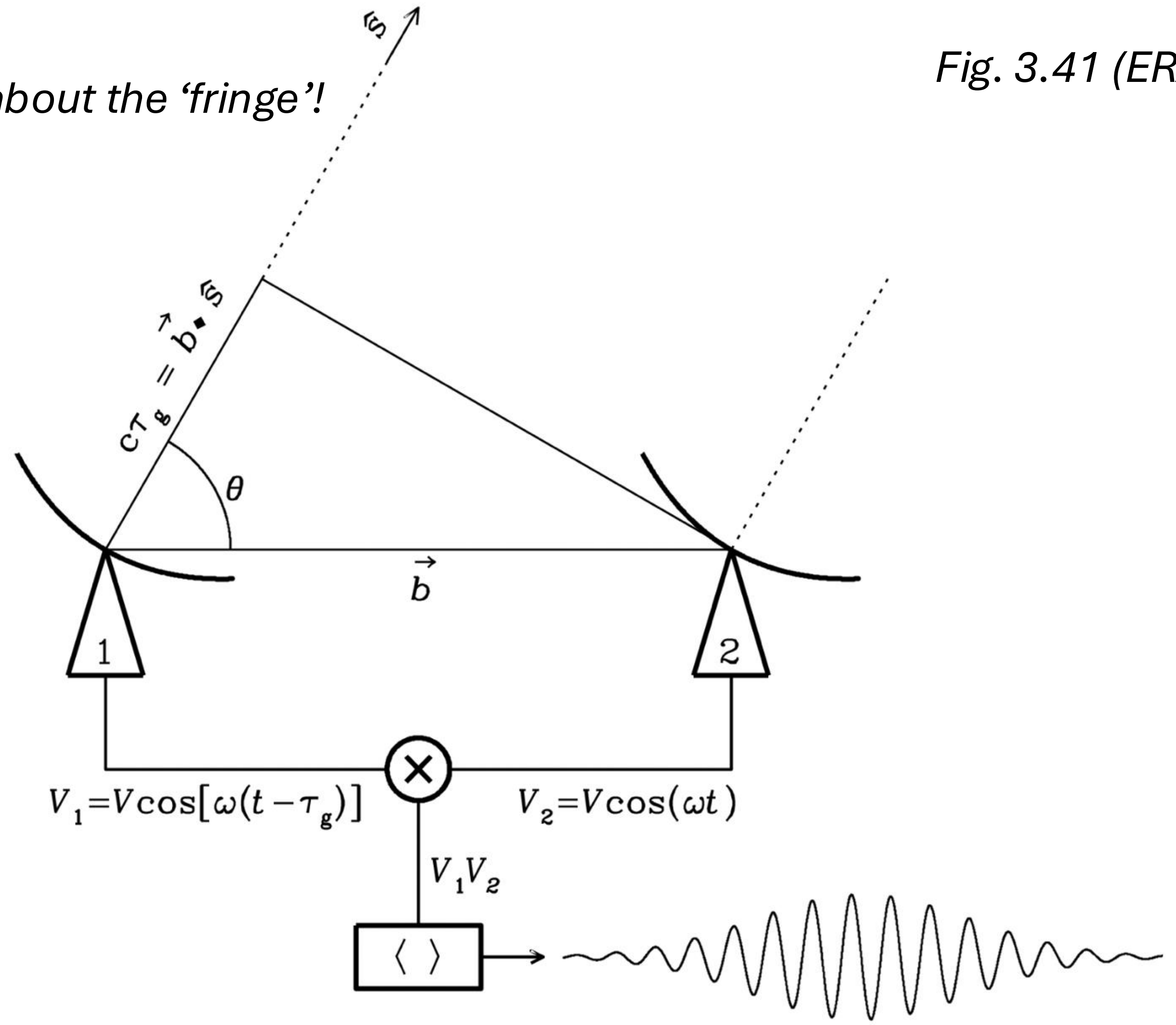

Output: $R = (V^2/2)\cos(\omega\tau_g)$

In this setup, you **multiply** the voltage outputs V_1 and V_2 **directly with correlators**, whose output is the measure of how alike the signals received by the two antennas are when one takes into account the time delay between them.

Remember:

$$\tau_g = \frac{\vec{b} \cdot \hat{s}}{c}. \qquad (3.172)$$

Where, $\vec{b} \cdot \hat{s} = b\cos\theta$

Multiplying or Coherent Interferometers:

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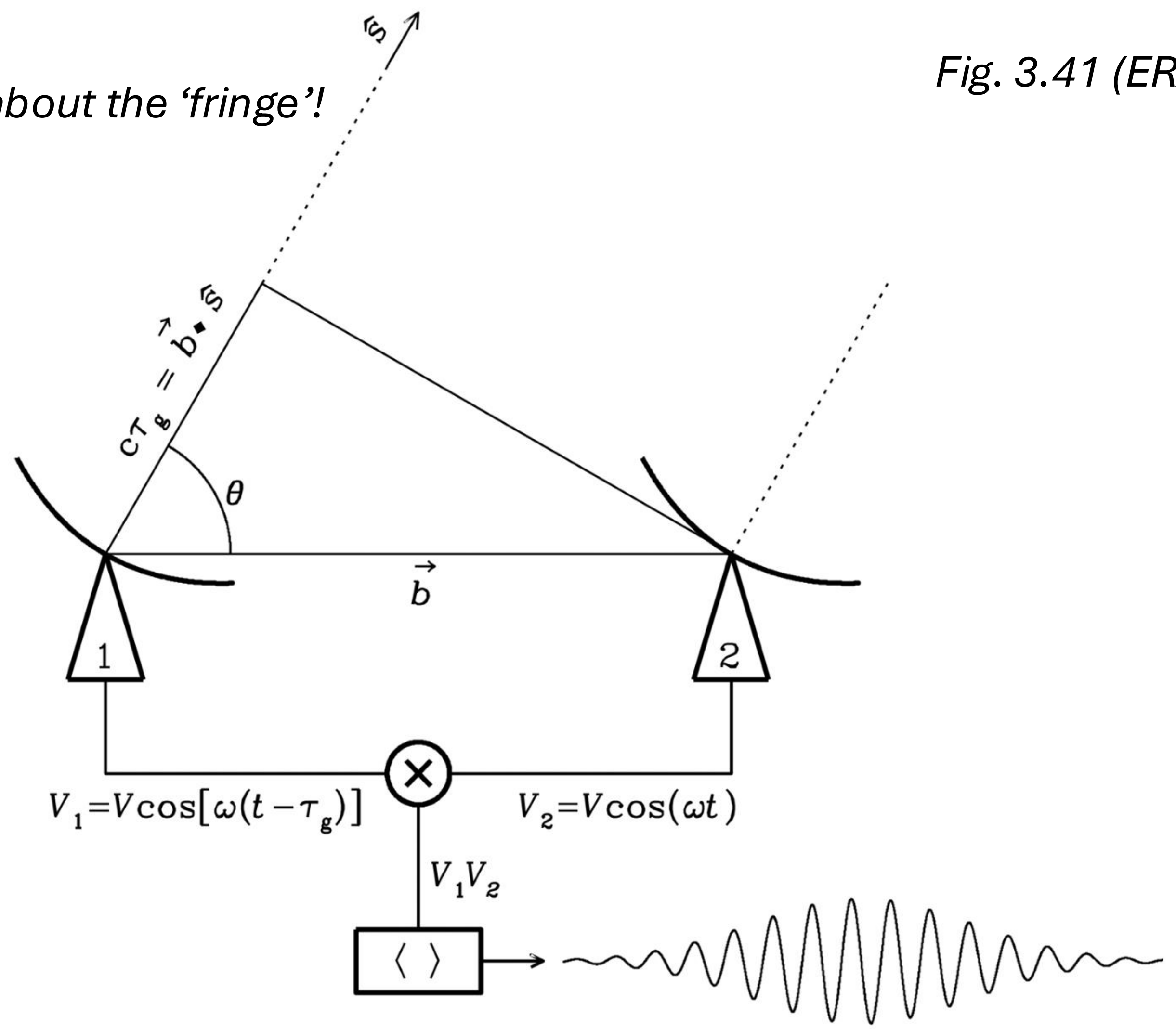

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The correlator output voltage, *R*, varies sinusoidally in the **fringe patterns** we are familiar with, where the fringe phase is,

$$\phi = \omega\tau_g = \frac{\omega}{c} b\cos\theta \qquad (3.176)$$

And depends on *θ*,

$$\frac{d\phi}{d\theta} = -\frac{\omega}{c} b\sin\theta \qquad (3.177)$$

$$= -2\pi\left(\frac{b\sin\theta}{\lambda}\right). \qquad (3.178)$$

** The fringe period, $\Delta\phi = 2\pi$, corresponds to the angular shift, $\Delta\theta = \lambda/(b\sin\theta)$*

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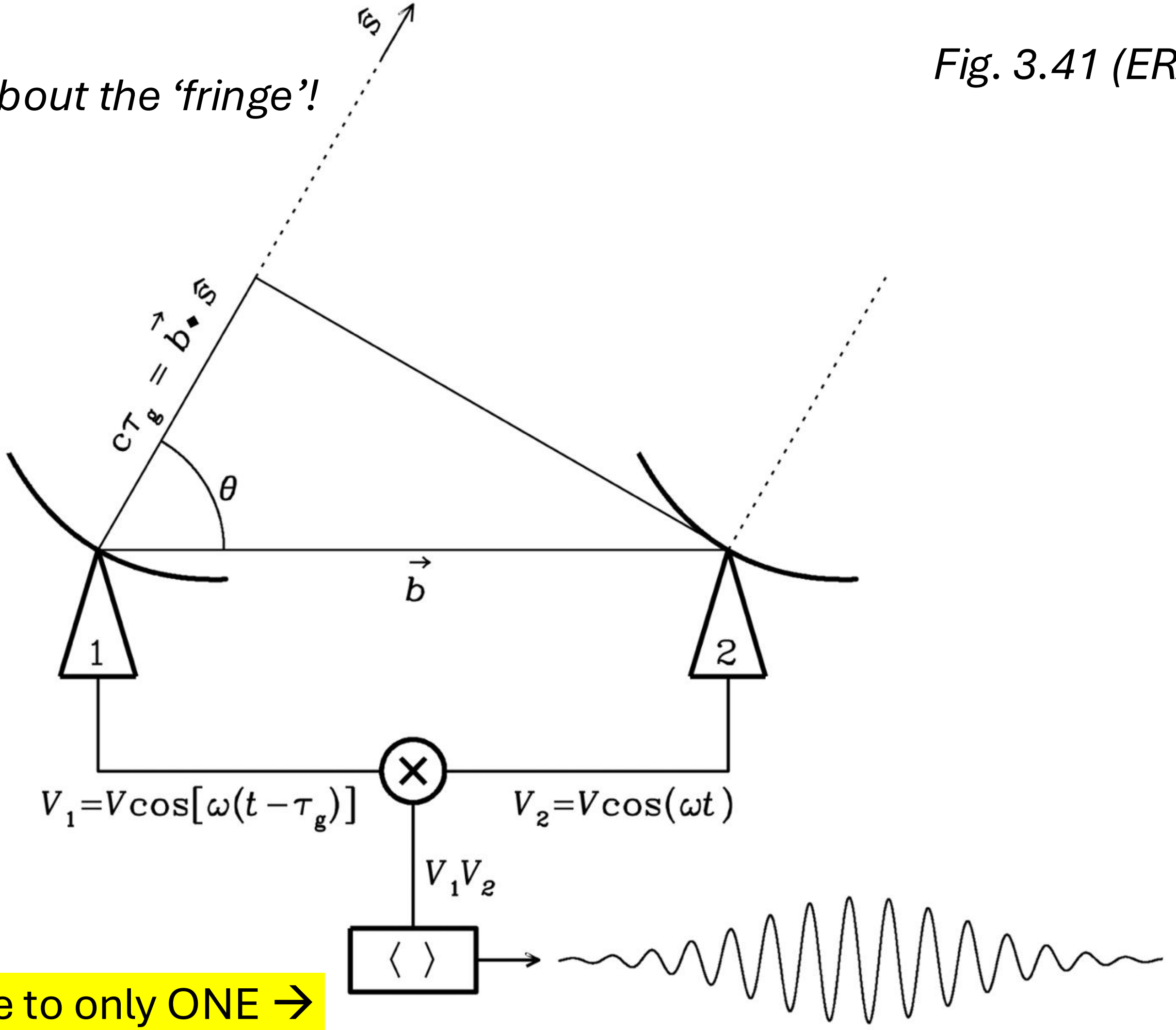

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** The fringe period, $\Delta\phi = 2\pi$, corresponds to the angular shift, $\Delta\theta = \lambda / (b\sin\theta)$*

Again, the two-component interferometer here is sensitive to only ONE → Fourier component of the sky brightness distribution

Output: $R = (V^2/2)\cos(\omega\tau_g)$

Multiplying or Coherent Interferometers:

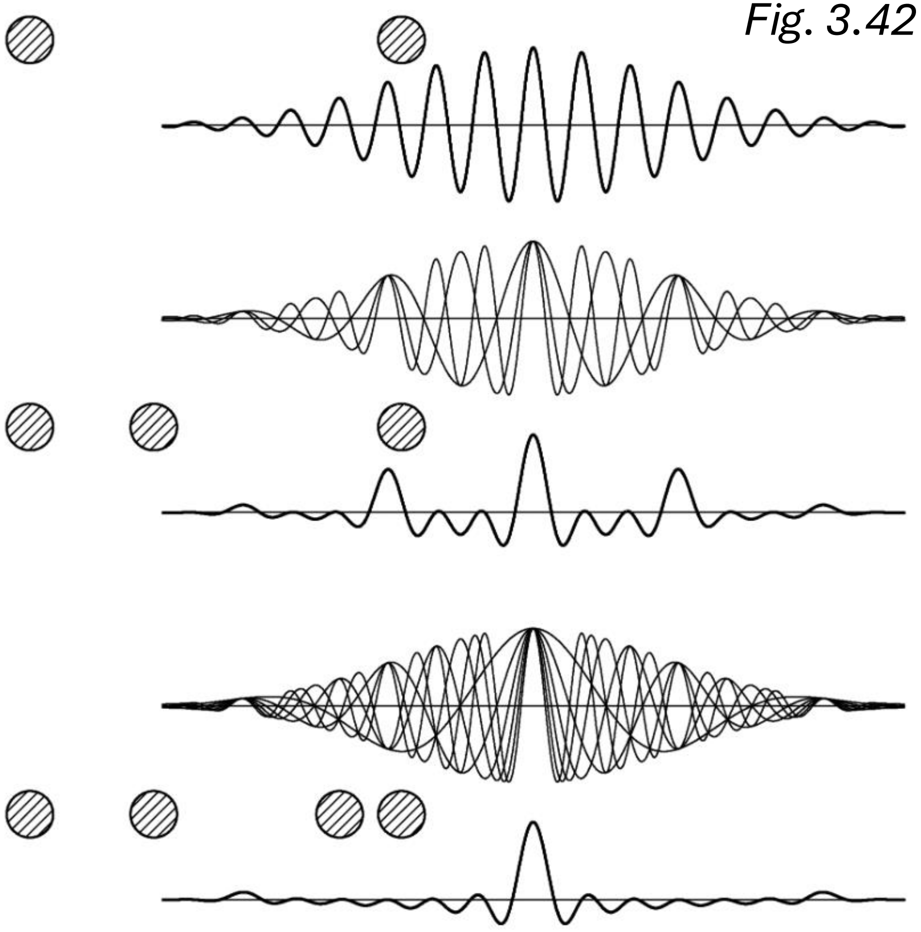
It's still all about the 'fringe'!

We need more baselines!

- AKA more **fringes!**
- This provides more Fourier components so that the beam can be **synthesized**, i.e., averaging the outputs of all the two-element interferometers to a point source response →
- This **synthesized beam** is referred to as the **dirty beam**

- Negative 'bowl' observed, so as we say before, we are not fully resolving the full image due to lack of samples at small baselines known as 'short spacing' (need single dish to recover!)

Fig. 3.42 (ERA)



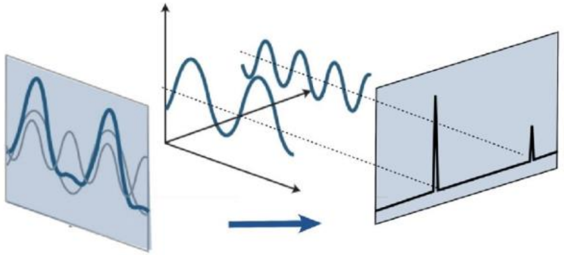
Angular resolution: λ/b

The Complex Correlator & Mapping the True Sky Brightness

To image the true sky brightness distribution, $I_\nu(s)$ near the frequency ν = ω/(2π), the emission is **treated as the sum of independent point sources**, where the two-element interferometer **response** (output voltage**)** or **fringe pattern** is given as,

$$R_c = \int I(\hat{s}) \cos(2\pi\nu \vec{b} \cdot \hat{s}/c)\, d\Omega = \int I(\hat{s}) \cos(2\pi \vec{b} \cdot \hat{s}/\lambda)\, d\Omega. \qquad (3.179)$$

Fourier Transform:



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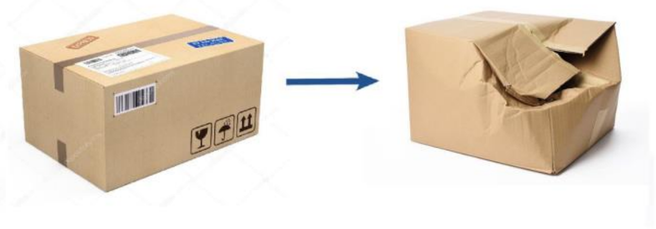

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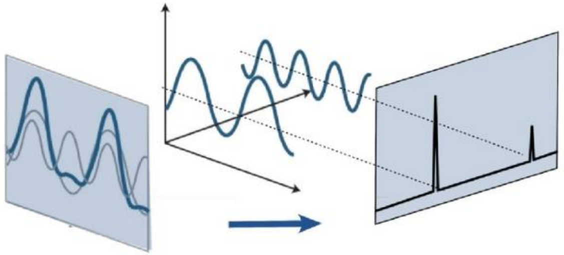
Remember our Fourier transform lecture! Cosine is an even function and only sensitive to symmetric parts… need a second 'sine' correlator to get odd part!

$$R_s = \int I(\hat{s}) \sin(2\pi \vec{b} \cdot \hat{s}/\lambda) d\Omega. \qquad (3.180)$$

Fourier Transform:



Courier Transform:

Euler's formula:

$$e^{i\phi} = \cos\phi + i\sin\phi,$$

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$$R_s = \int I(\hat{s}) \sin(2\pi \vec{b} \cdot \hat{s}/\lambda)d\Omega. \qquad (3.180)$$

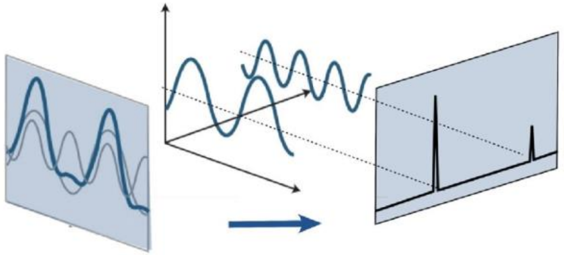
Combining the response from both the cosine and sine correlators is a 'complex correlator' and we can then define our 'fringes' again by this term **'visibility'** where the **complex visibility** is defined as:

$$\mathscr{V} \equiv R_c - iR_s \qquad (3.181)$$

which becomes

$$\mathscr{V} = Ae^{-i\phi}, \qquad (3.182)$$

Fourier Transform:



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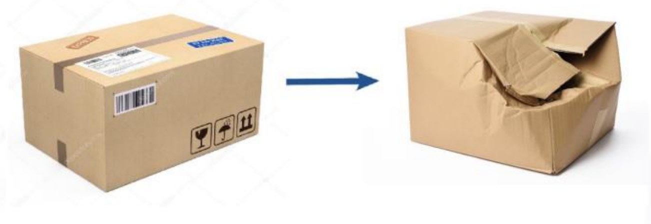
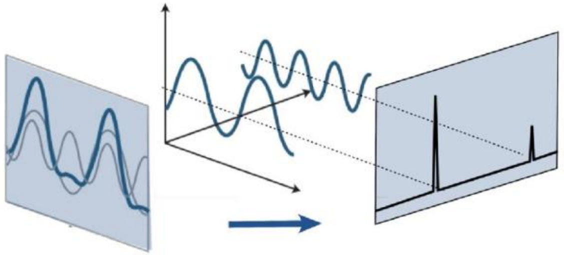
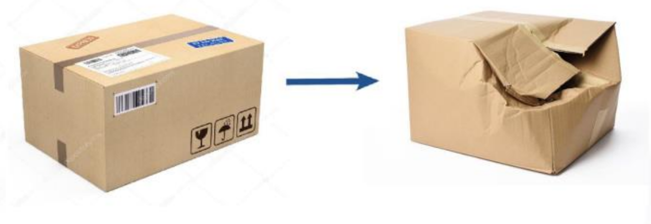


Euler's formula:

$$e^{i\phi} = \cos\phi + i\sin\phi,$$

The Complex Correlator & Mapping the True Sky Brightness





To image the true sky brightness distribution, $I_\nu(s)$ near the frequency ν = ω/(2π), the emission is **treated as the sum of independent point sources**, where the two-element interferometer **response** (output voltage) or **fringe pattern** is given as,

$$R_c = \int I(\hat{s}) \cos(2\pi\nu \vec{b} \cdot \hat{s}/c)\, d\Omega = \int I(\hat{s}) \cos(2\pi \vec{b} \cdot \hat{s}/\lambda)\, d\Omega. \qquad (3.179)$$

Remember our Fourier transform lecture! Cosine is an even function and only sensitive to symmetric parts… need a second 'sine' correlator to get odd part!

$$R_s = \int I(\hat{s}) \sin(2\pi \vec{b} \cdot \hat{s}/\lambda)\, d\Omega. \qquad (3.180)$$

Combining the response from both the cosine and sine correlators is a 'complex correlator' and we can then define our 'fringes' again by this term **'visibility'** where the **complex visibility** is defined as:

$$\mathscr{V} \equiv R_c - iR_s \qquad (3.182)$$

which becomes

$$\mathscr{V} = Ae^{-i\phi}, \qquad (3.183)$$

This is what your interferometer actually measures! Just Fourier transform to get  sky brightness!

The Complex Correlator & Mapping the True Sky Brightness

$$\mathscr{V} = Ae^{-i\phi}, \qquad (3.183)$$

Each visibility has an **amplitude (*A*)** and **phase (*ϕ*)** →

$$A = (R_c^2 + R_s^2)^{1/2} \qquad (3.184)$$

$$\phi = \tan^{-1}\ (R_s/R_c) \qquad (3.185)$$

Amplitude tells how bright it is
Phase where it is

The response to an extended source with brightness distribution $I_\nu(s)$ of the two-element quasi-monochromatic interferometer with a complex correlator is the complex visibility is thus,

$$\mathscr{V} = \int I(s)\exp(-i2\pi \vec{b} \cdot s/\lambda)\ \ d\Omega. \qquad (3.186)$$

The Complex Correlator & Mapping the True Sky Brightness

Typically, you will work in the 'u,v' plane to define your visibilities:

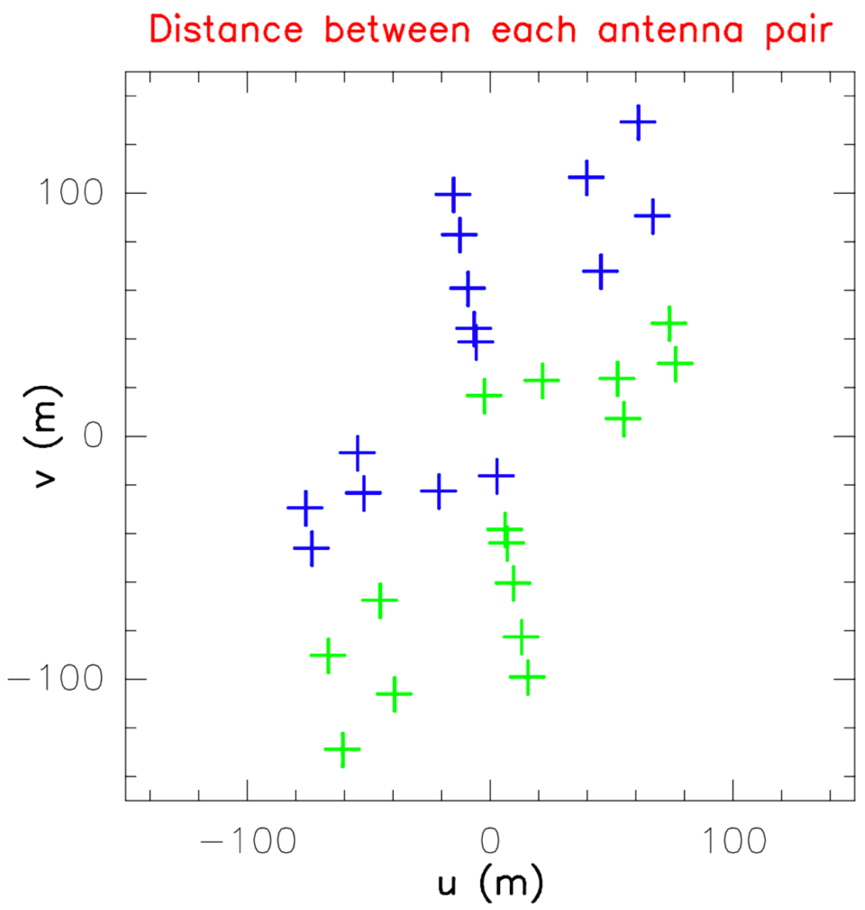
Fourier transform*

$$V(u,v) \underset{\textbf{FT}}{\rightarrow} T(x,y)$$

V(u,v) = the complex visibility function $= \iint T(x,y)e^{2\pi i(ux+vy)}dxdy$

T(x,y) = the sky brightness distribution $= \iint V(u,v)e^{-2\pi i(ux+vy)}dudv$

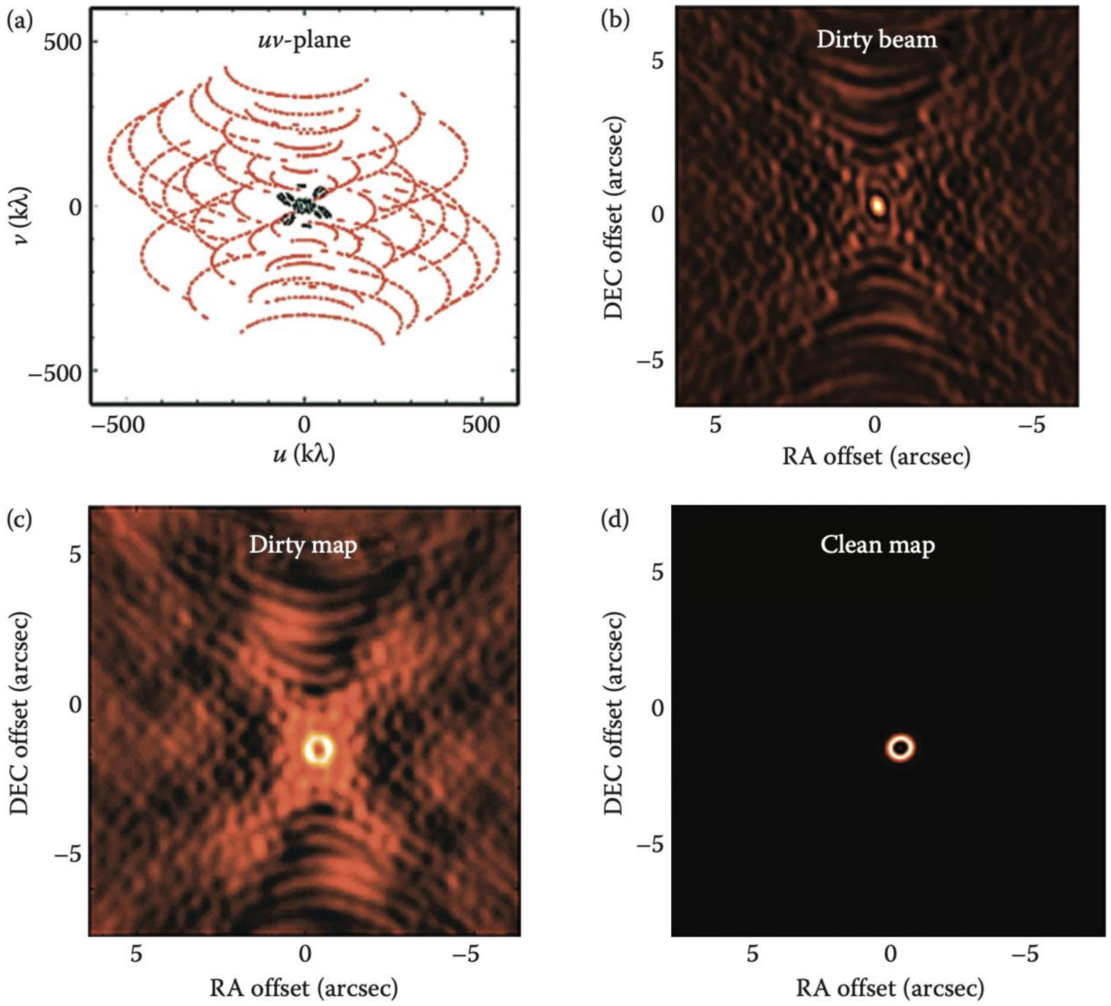
Where each pair of antenna (i.e. baseline) corresponds to discrete points on the UV plane.



Slide Credit: ALMA Ambassadors Workshop

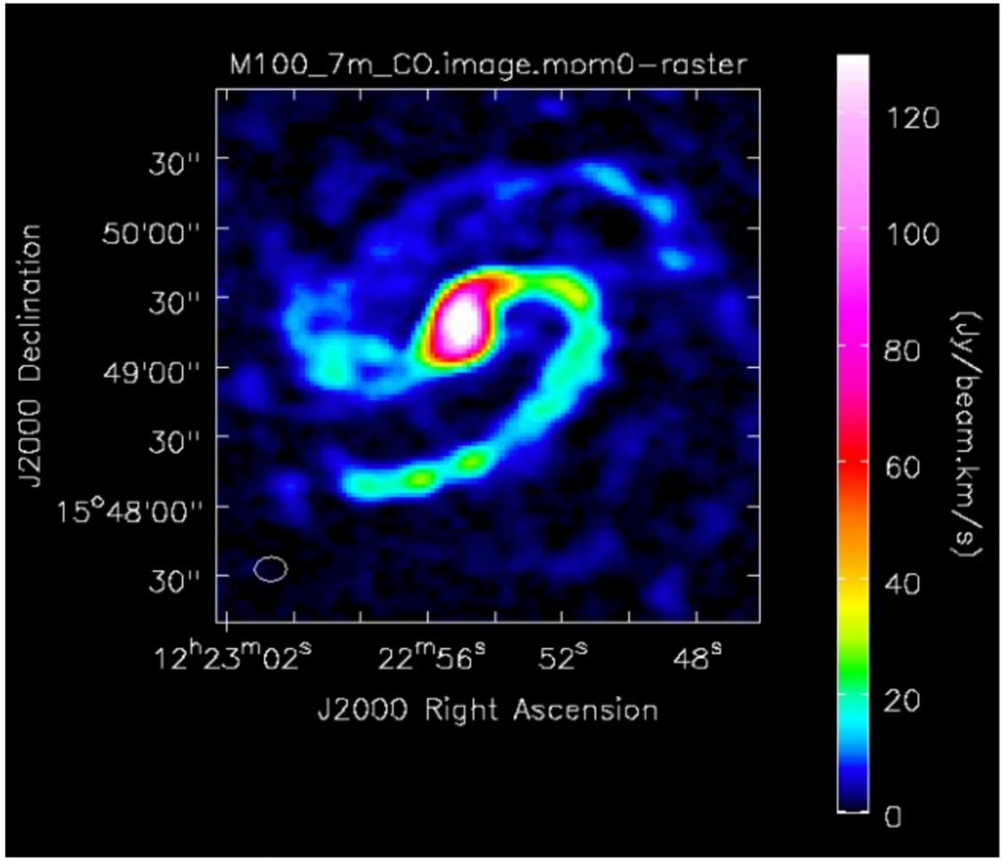
The Complex Correlator & Mapping the True Sky Brightness

- Even if the uv-space is under sampled, an observed source intensity distribution can be computed for any values of x, y
- The more telescopes there are making up the interferometer, the greater the number of baseline pairs, and the faster the *uv*-plane can be filled in
- Missing pieces of the large telescope mean that the synthesized beam is incomplete or ‘dirty’
- A ‘dirty map’ is a source image made using a dirty beam
- A ‘clean map’ is produced by filling in missing parts of the uv-plane with best guess estimates

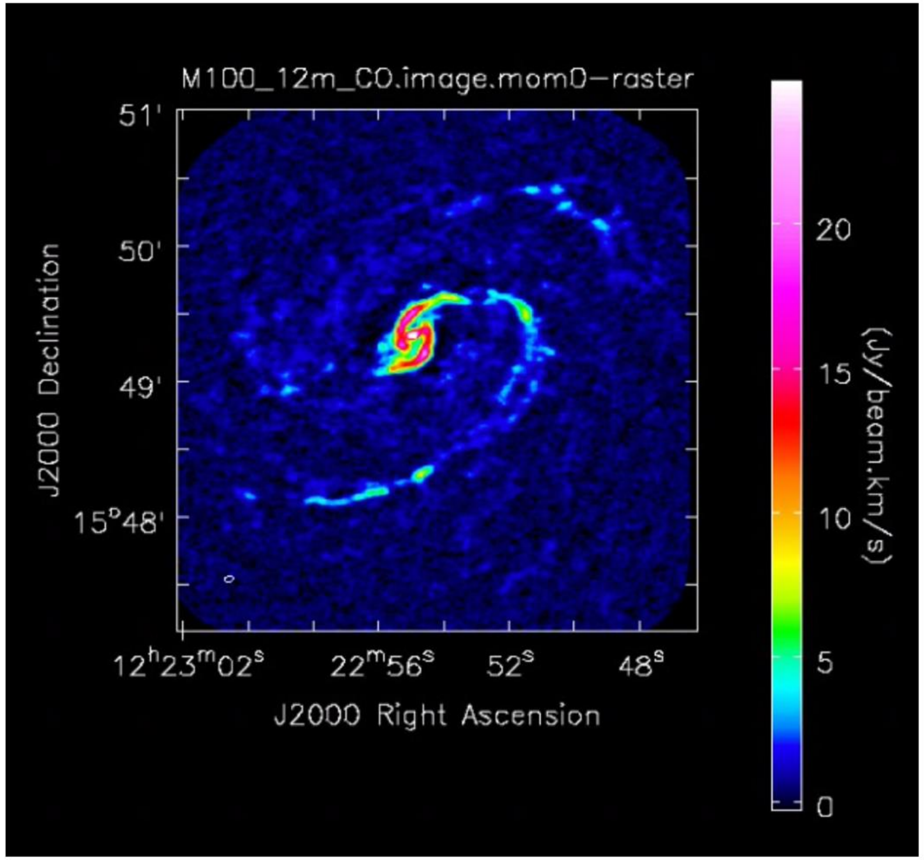


The Complex Correlator & Mapping the True Sky Brightness

Short baselines (i.e. missing long baselines)

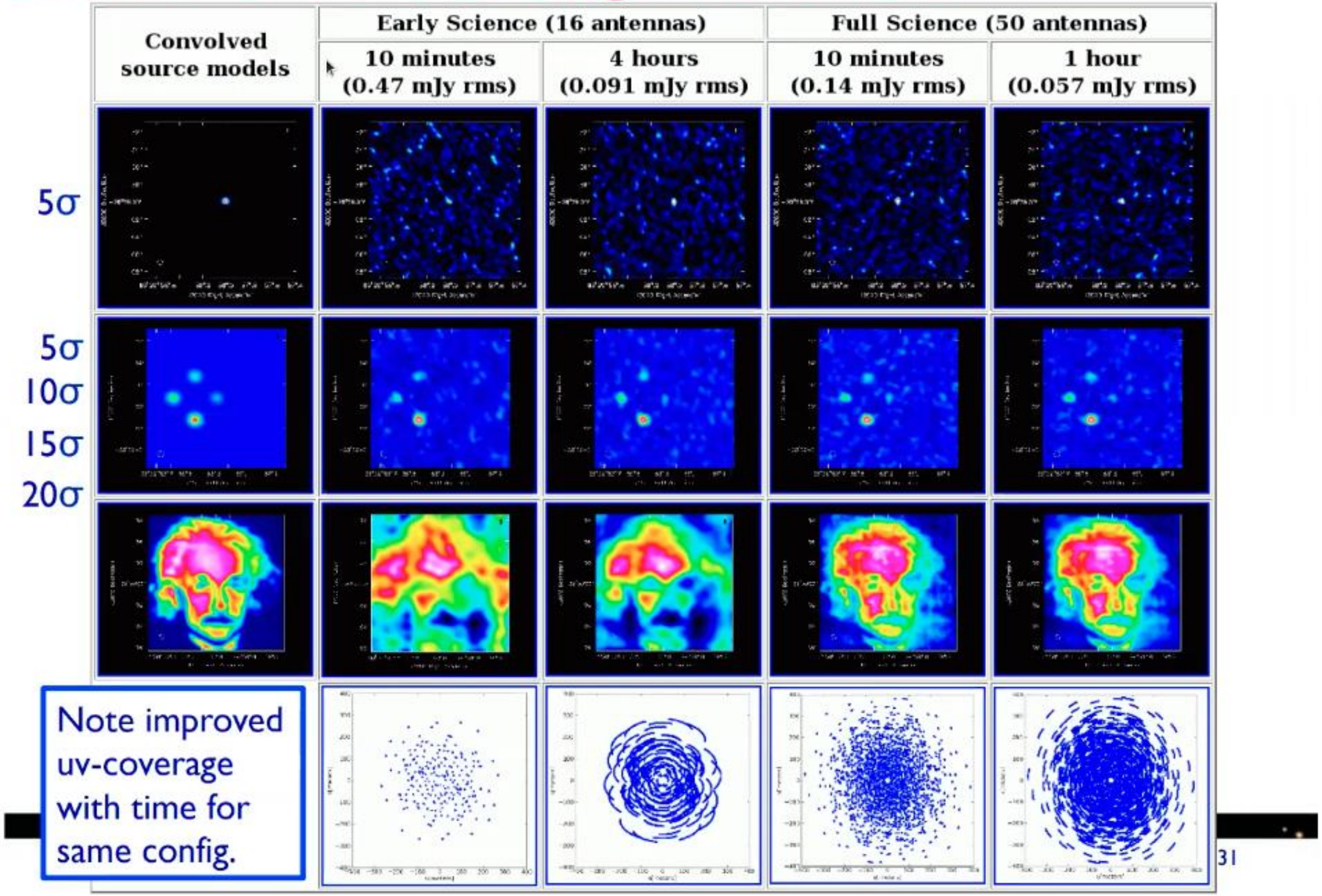


Long baselines (i.e. missing short baselines)

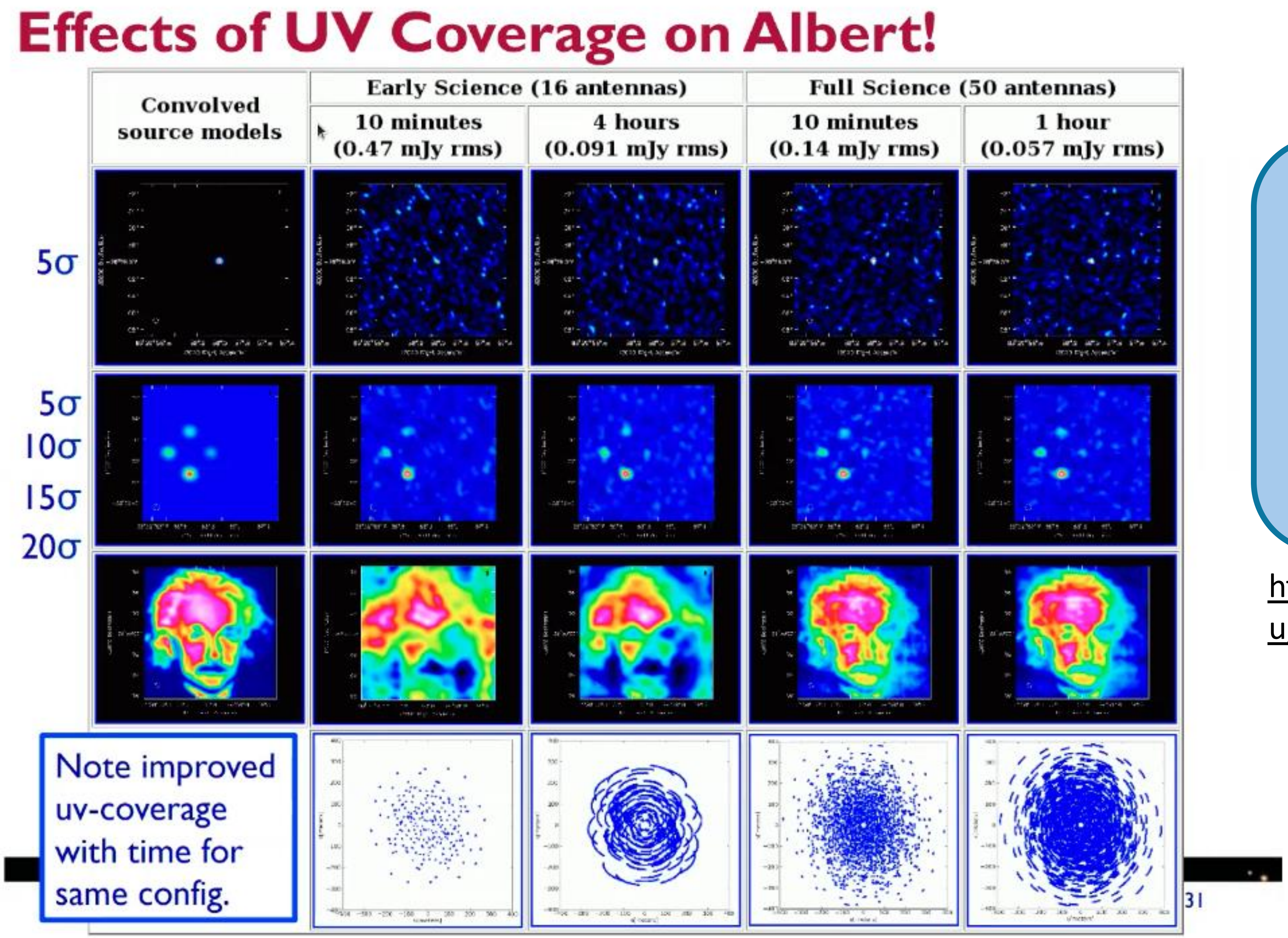


Slide Credit: ALMA Ambassadors Workshop (Dominic Ludovici)

Effects of UV Coverage on Albert!



Slide Credit: NRAO Synthesis Imaging Workshop



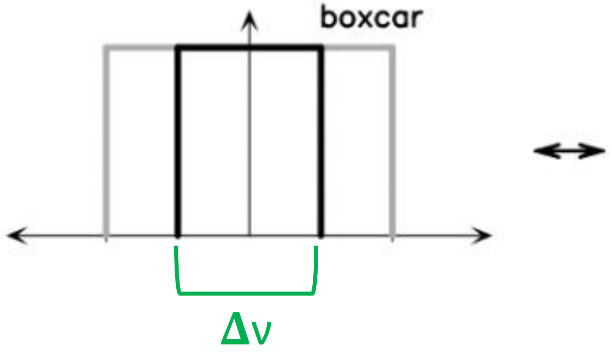
Check out online tool - let's you 'build your own interferometer'! + watch Urvashi's lecture if you haven't yet!

https://github.com/urvashirau/ImagingSimulator

Slide Credit: NRAO Synthesis Imaging Workshop

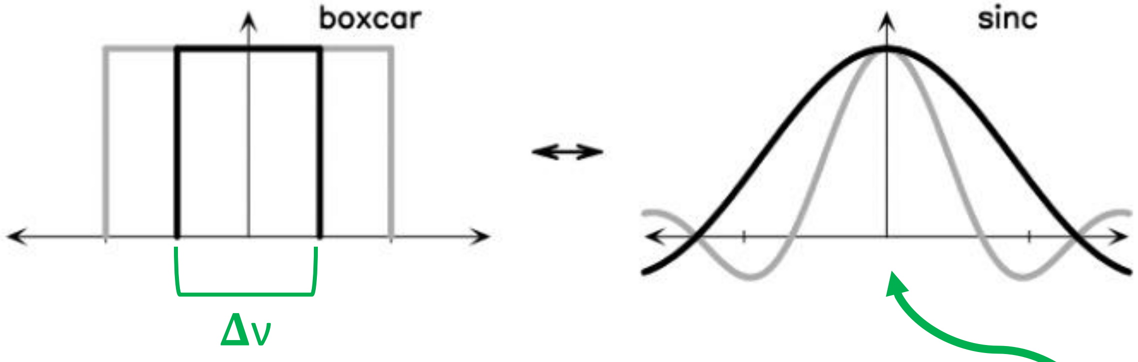
Effects of Finite Bandwidths and Averaging Times

1st consider you are observing in a **finite frequency range**, Δν, and the source brightness and response of your interferometer is nearly constant – you can consider it a box function:



Effects of Finite Bandwidths and Averaging Times

1st consider you are observing in a **finite frequency range**, $\Delta\nu$, and the source brightness and response of your interferometer is nearly constant – you can consider it a box function:



The **fringe amplitude** is therefore attenuated by the factor sinc($\Delta\nu\tau_g$)

Effects of Finite Bandwidths and Averaging Times

Fig. 3.43 (ERA)

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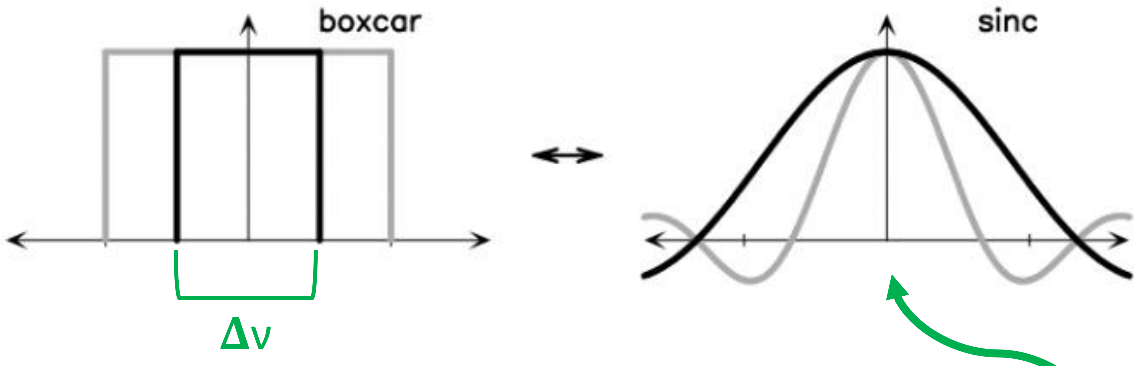
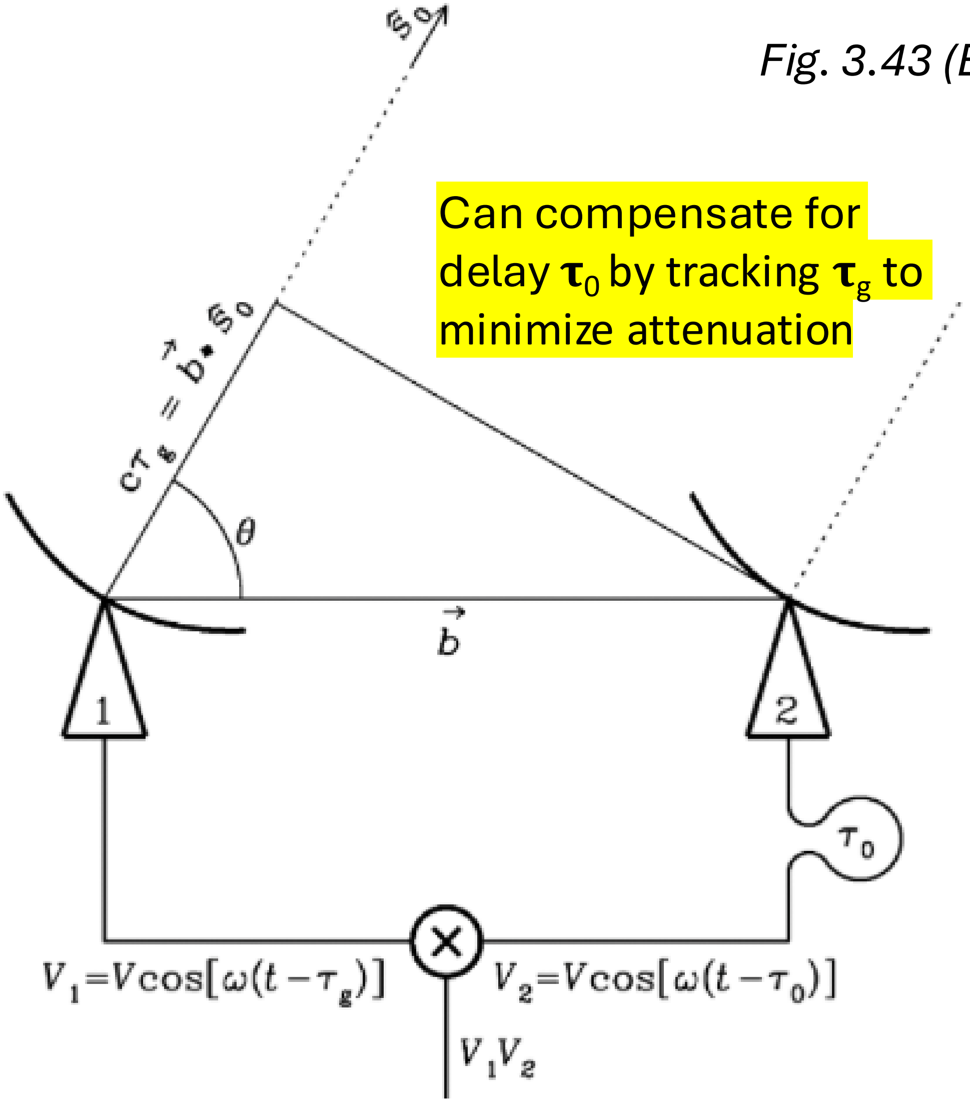


The **fringe amplitude** is therefore attenuated by the factor sinc($\Delta\nu\tau_g$)

Can compensate for delay τ_0 by tracking τ_g to minimize attenuation



Effects of Finite Bandwidths and Averaging Times

Following some algebra in the book, the important requirement to remember is:

Angular FOV radius / resolution

$$\boxed{\frac{\Delta\theta}{\theta_s} \ll \frac{\nu}{\Delta\nu}.} \quad (3.192)$$

Frequency / bandwidth

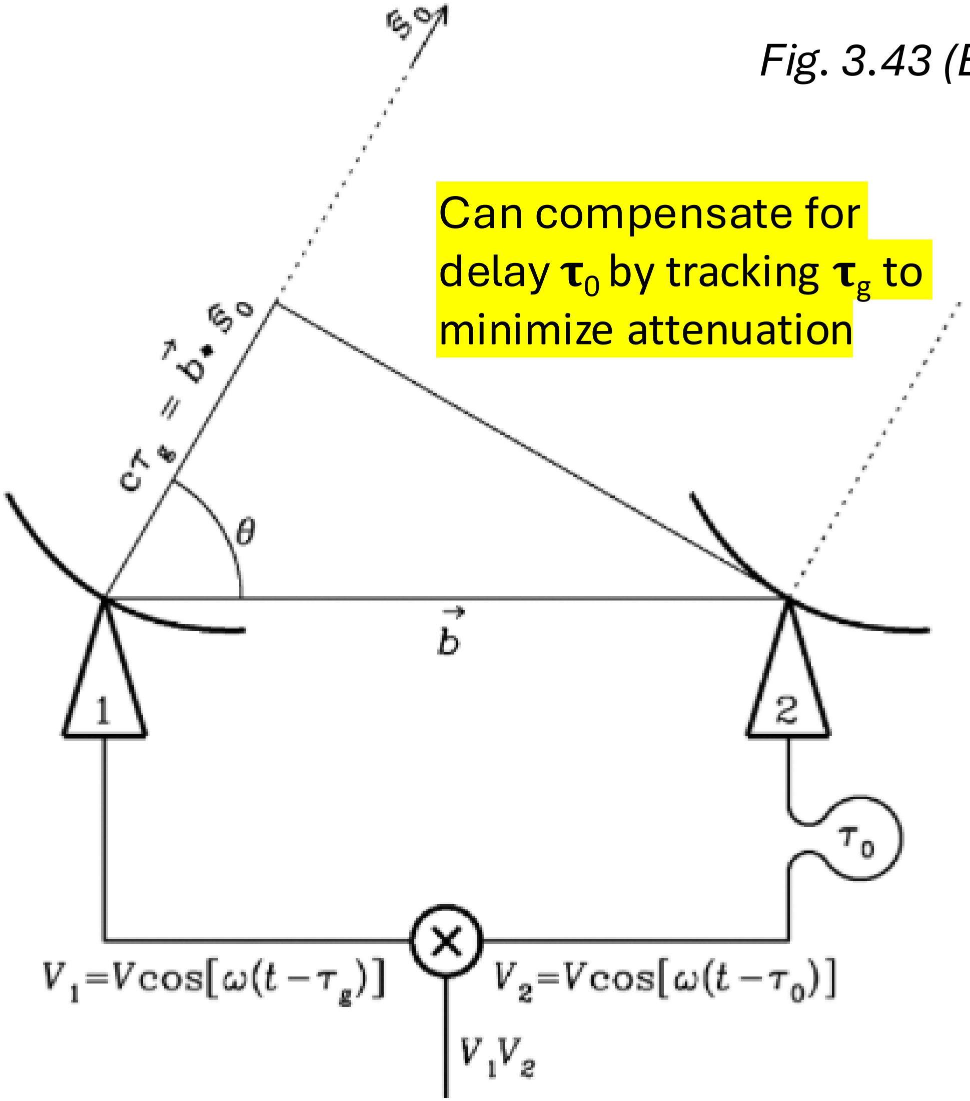
!Important! This limits the **bandwidth** to make an image. If not followed there can be **bandwidth smearing** that can broaden the synthesized beam

And, if correlating averaging times are too large this will cause **time smearing** and the correlator average time must follow:

$$\boxed{\frac{2\pi\Delta t}{P} \approx \frac{\Delta t}{1.37 \times 10^4\ \text{s}} \ll \frac{\theta_s}{\Delta\theta}.} \quad (3.194)$$

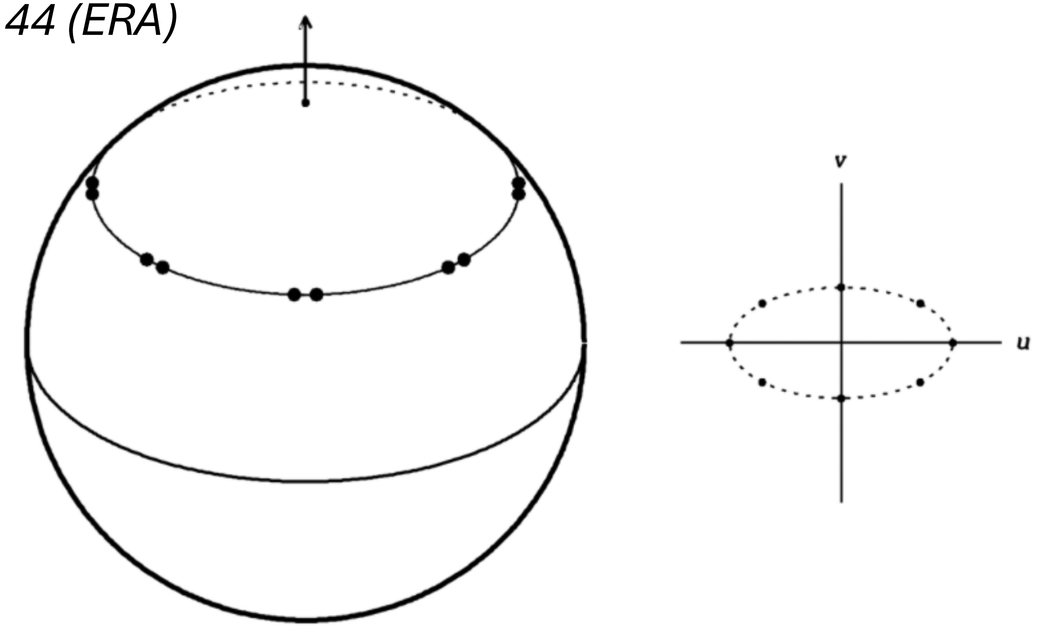
Fig. 3.43 (ERA)

Can compensate for delay $\boldsymbol{\tau}_0$ by tracking $\boldsymbol{\tau}_g$ to minimize attenuation



Earth-Rotation Aperture Synthesis

Fig. 3.44 (ERA)



Ideally, if your baselines are confined to the east-west line in a single plane perpendicular to Earth's north-south rotation axis your brightness distribution is a simple 2D Fourier transform

*The natural rotation of the earth will help to fill-in the (u, v) plane

Earth-Rotation Aperture Synthesis

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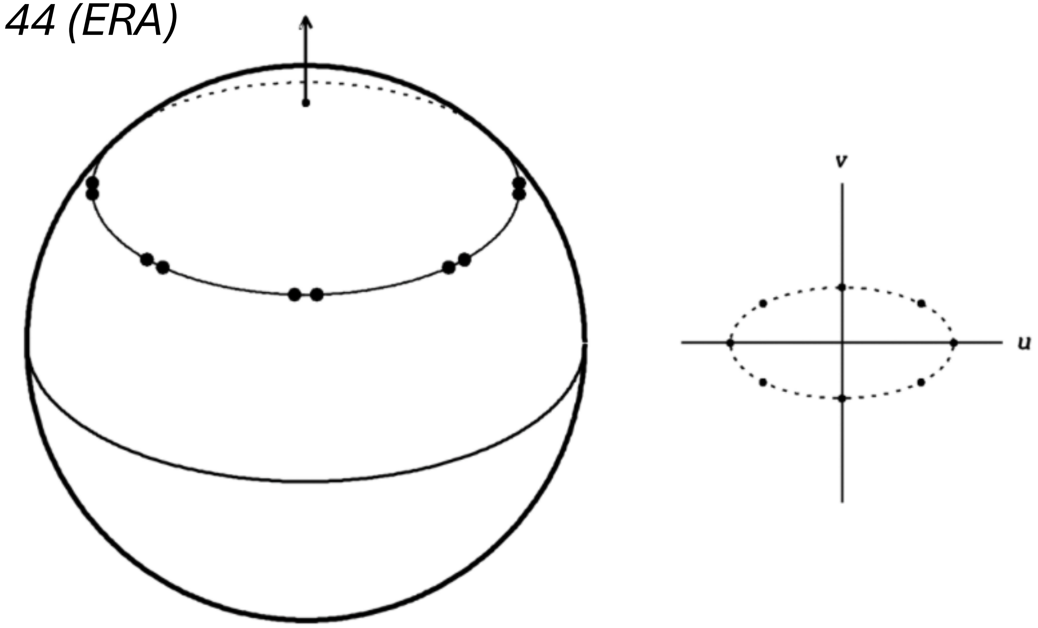


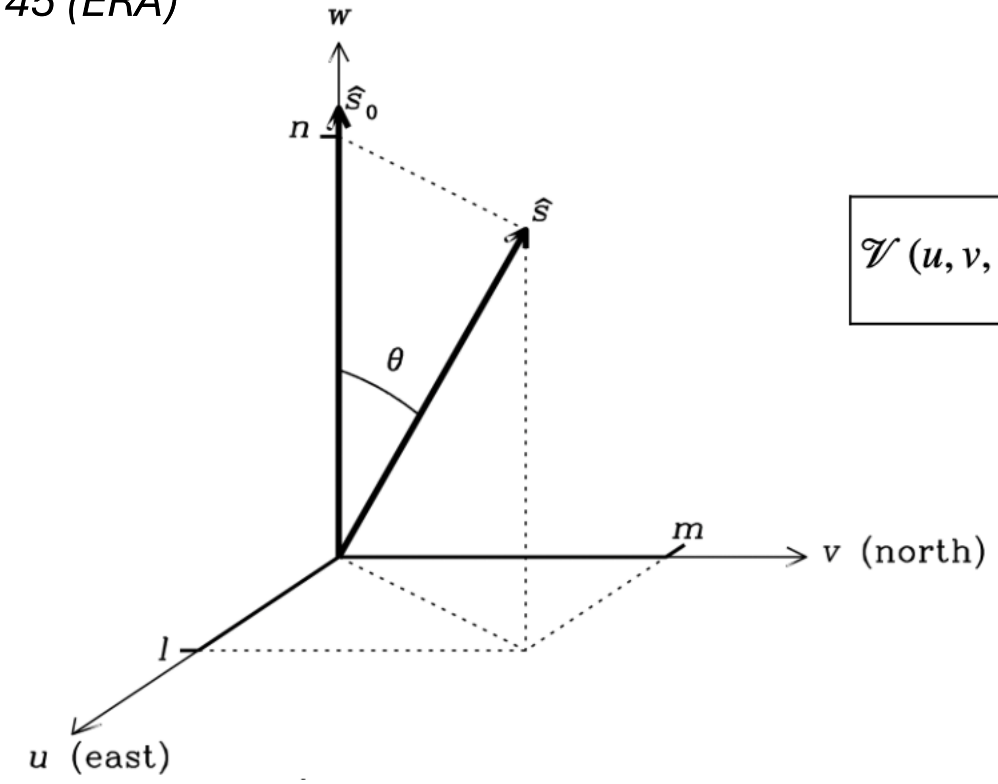
Ideally, if your baselines are confined to the east-west line in a single plane perpendicular to Earth's north-south rotation axis your brightness distribution is a simple 2D Fourier transform

*The natural rotation of the earth will help to fill-in the (u, v) plane

Issue! Most interferometers and baselines are not confined to an east-west line (e.g., the VLA) and on long timescales Earth's rotation cause the VLA baselines to fill a 3D volume

How do we deal with 3D plane?

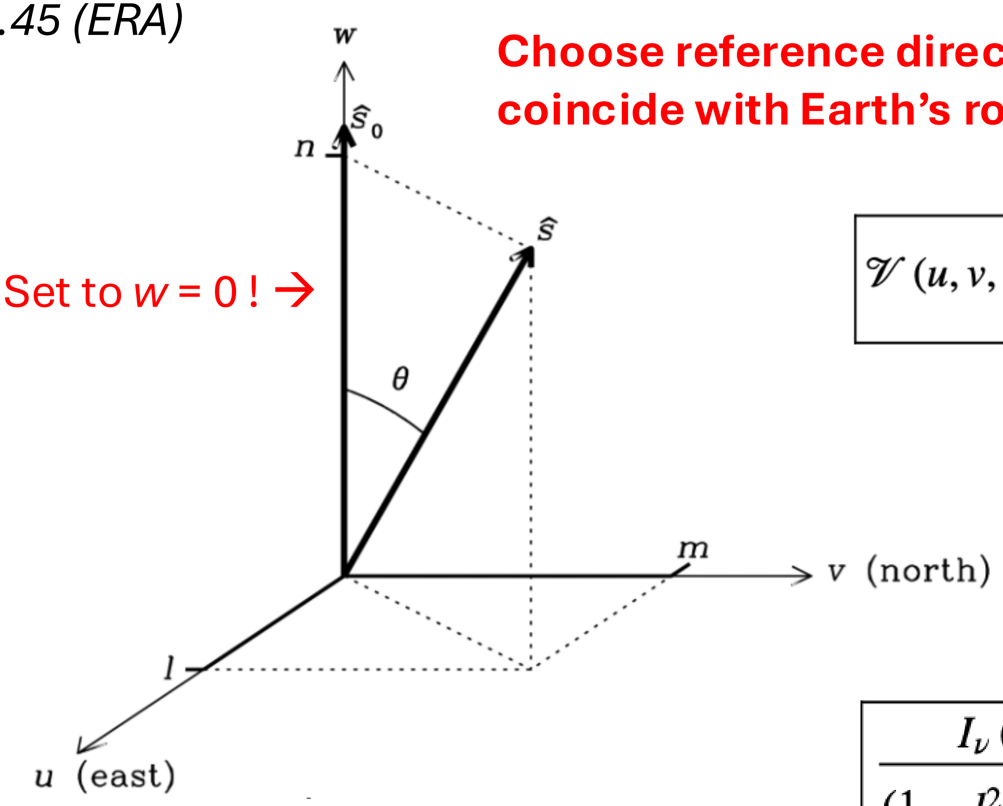
Fig. 3.45 (ERA)



$$\mathscr{V}(u,v,w) = \int\int \frac{I_\nu(l,m)}{(1-l^2-m^2)^{1/2}} \exp\left[-i2\pi(ul+vm+wn)\right] dl\,dm. \quad (3.197)$$

How do we deal with 3D plane?

Fig. 3.45 (ERA)



Choose reference direction to be coincide with Earth's rotation axis

Set to $w = 0$! →

$$\mathscr{V}(u, v, w) = \int\int \frac{I_\nu(l, m)}{(1 - l^2 - m^2)^{1/2}} \exp\left[-i2\pi(ul + vm + wn)\right] dl\, dm. \quad (3.197)$$

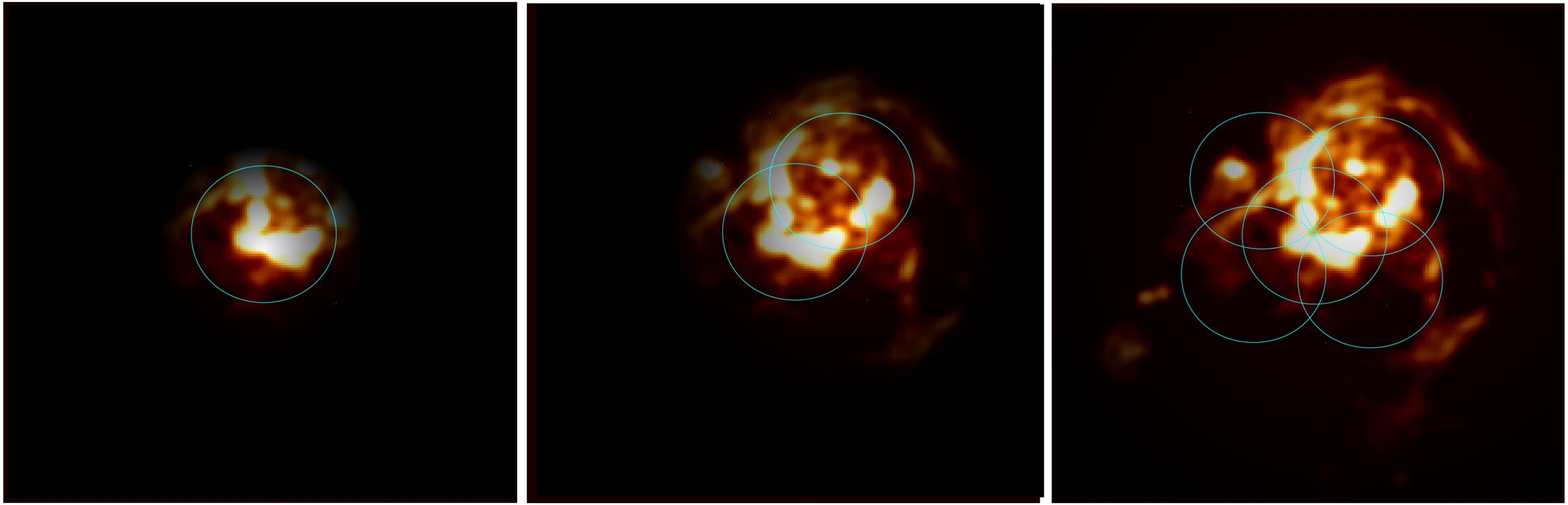
NOT a Fourier transform!

If we set $w = 0$, it becomes a 2D Fourier transform that can be inverted to give the source brightness in terms of measured visibilities:

$$\frac{I_\nu(l, m)}{(1 - l^2 - m^2)^{1/2}} = \int\int \mathscr{V}(u, v, 0) \exp\left[+i2\pi(ul + vm)\right] du\, dv. \quad (3.198)$$

How do we deal with 3D plane?

Combine data from multiple pointings to form one large image.



Slide Credit: Urvashi for NRAO REU 2024

Sensitivity of an Interferometer

Point-source sensitivity for a single antenna,

$$\sigma_S = \frac{2kT_s}{A_e(\Delta\nu\,\tau)^{1/2}} \qquad (3.201)$$

And for a two-element interferometer (where A_e is the effective collecting area of each element):

$$\sigma_S = \frac{2^{1/2}kT_s}{A_e(\Delta\nu\,\tau)^{1/2}} \qquad (3.202)$$

The point-source sensitivity is $2^{1/2}$ times better than the sensitivity of each antenna, but $2^{1/2}$ times worse than that of a single dish whose area is that of two antennas

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Two antennas multiply two independent sets of voltages together to make a visibility and thus there are two independent sets of noise multiplied

Information contained in the two independent square-law detector outputs have been discarded – together they have $2^{1/2}$ times the sensitivity of a single dish

Combined with the independent correlator output, the total sensitivity is $(2 + 2)^{1/2}$ which is 2x the sensitivity of the single dish

Sensitivity of an Interferometer

Fig. 3.34 (ERA)

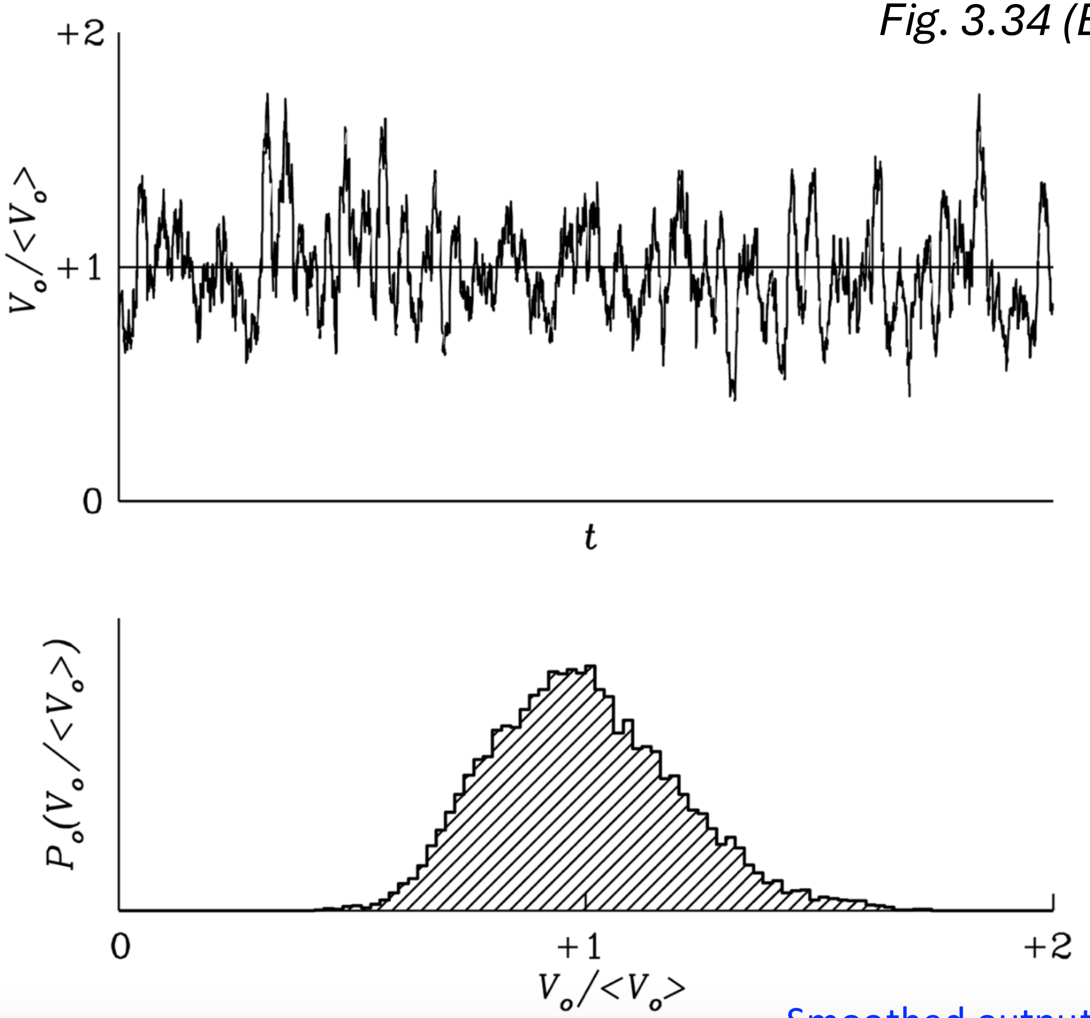
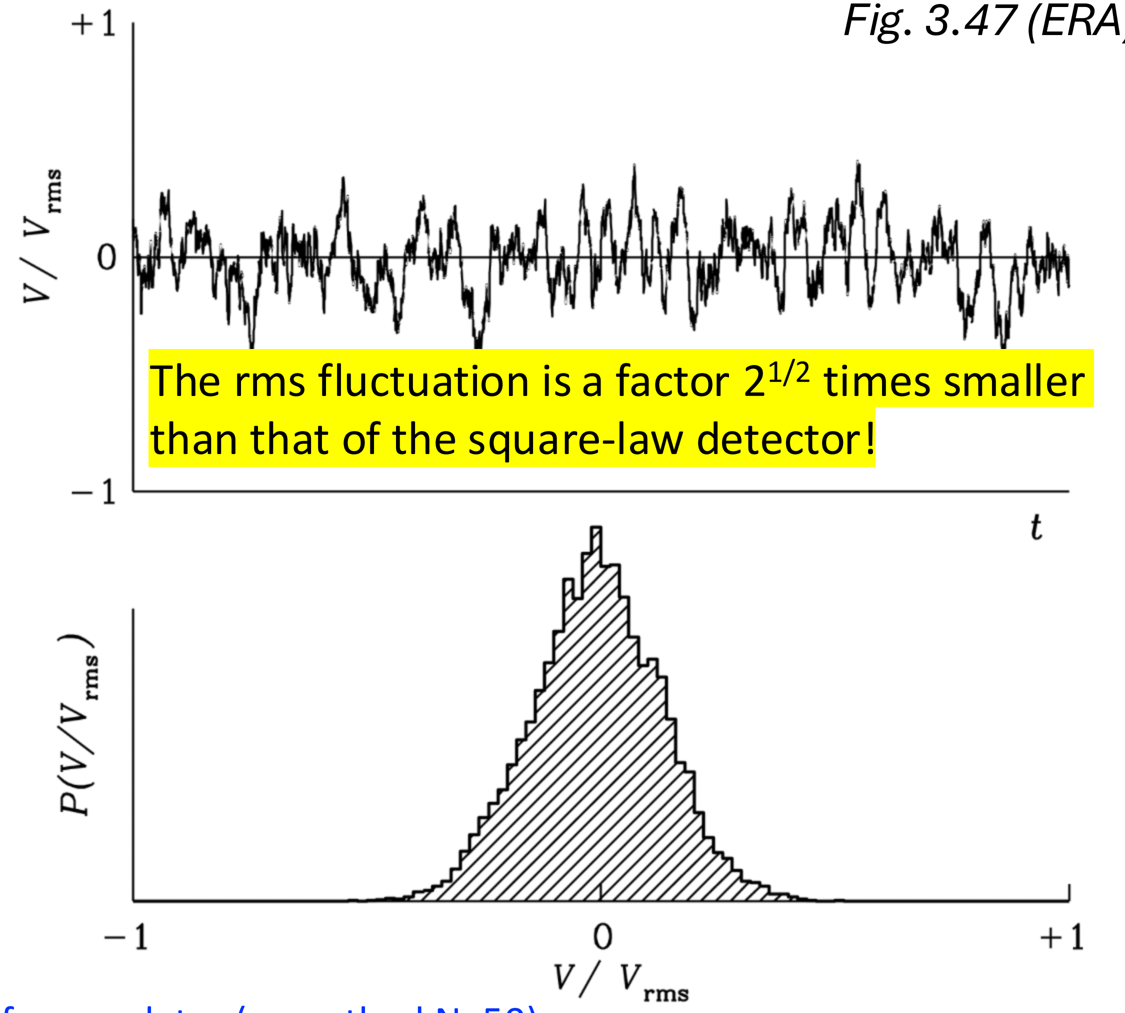


Fig. 3.47 (ERA)



The rms fluctuation is a factor $2^{1/2}$ times smaller than that of the square-law detector!

Smoothed output voltages of a correlator (smoothed N=50)

Sensitivity of an Interferometer

An interferometer with N dishes contains $N(N - 1)/2$ independent two-element interferometers. So long as the signal from each dish can be amplified *coherently* before it is spit up to be multiplied by the signals from the N-1 other antennas, its point-source rms noise (per beam) is

$$\sigma_S = \frac{2kT_s}{A_e[N(N-1)\,\Delta\nu\,\tau]^{1/2}}. \qquad (3.203)$$

The VLA with N = 27 dishes each d = 25 m in diameter has a sensitivity of a dish with $D = [N(N - 1)]^{1/4}\, d = [27(26)^{1/4}]$ 25 m = 129 m!

"Brightness" Sensitivity

BEWARE! The brightness sensitivity of an interferometer is **worse than a single dish** because the synthesized beam solid angle of an interferometer is much smaller than the beam solid angle of a single dish of the same total effective area

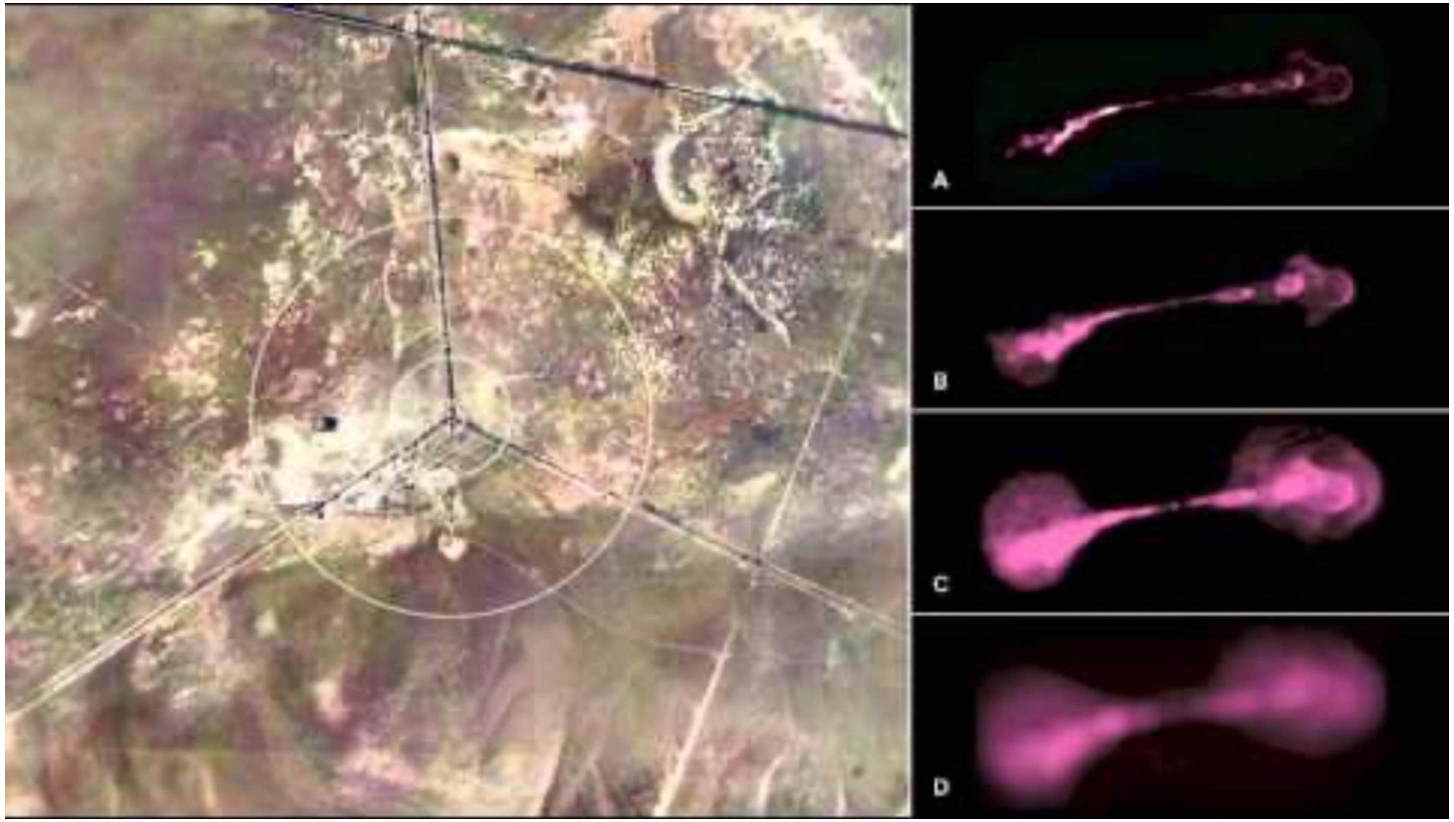
Interferometer resolution:

θ ~ λ/b radians

Single dish resolution:

θ ~ λ/D radians

Smaller by factor of ~ $(D/b)^2$ that defines the area **filling factor**



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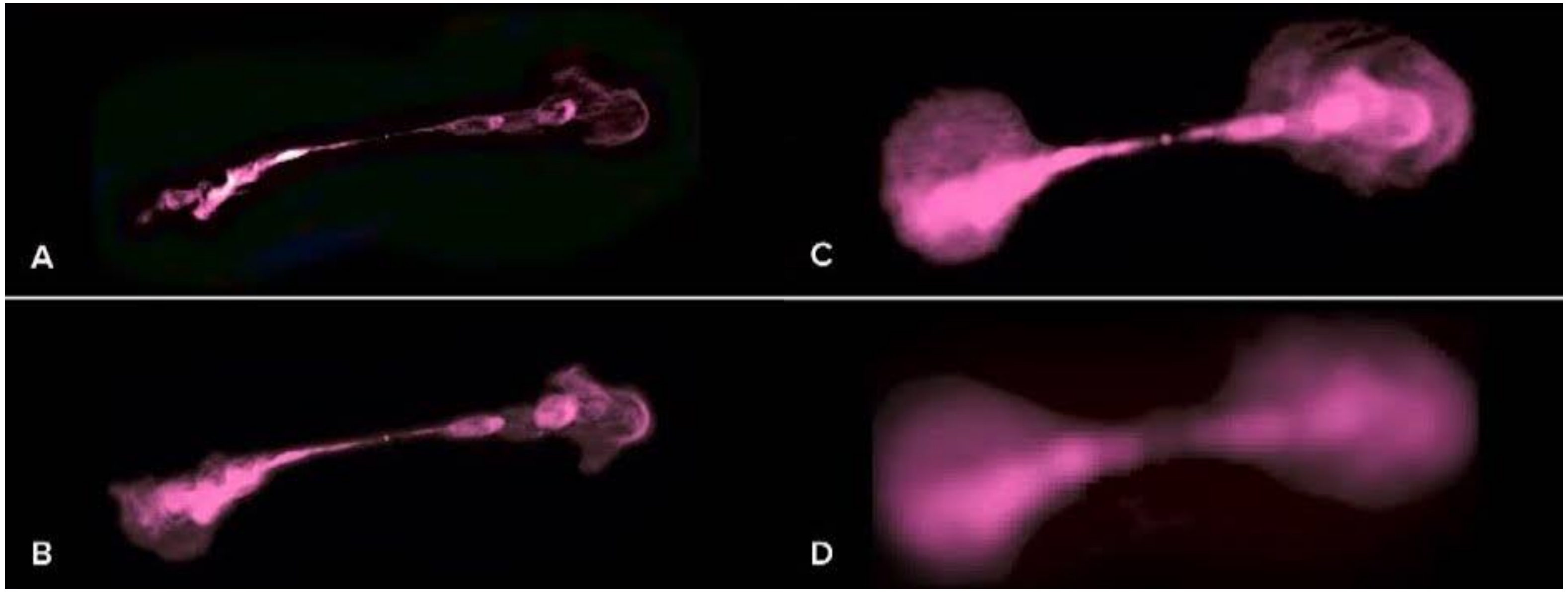


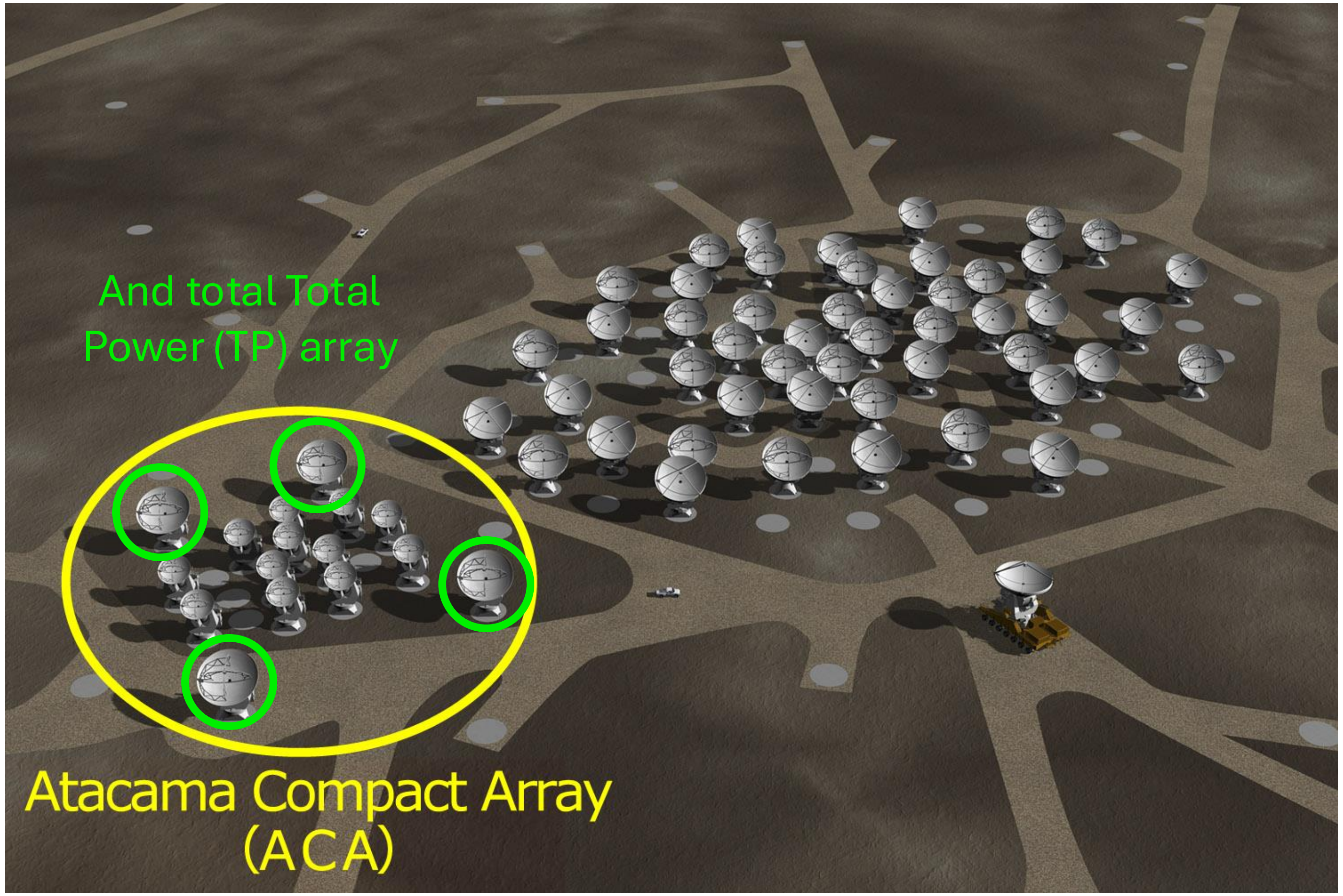
In order to be sensitive to emission at different scales, one often needs to **observe in different baseline configurations**

"Brightness" Sensitivity

ALMA tries to compensate to increase sensitivity and recover extended emission with, in addition to different 'configurations' of the 12m dishes:

- A compact 'ACA' setup with the use of 12 smaller 7m dishes
- A total power array made up of 4 of the 12m dishes

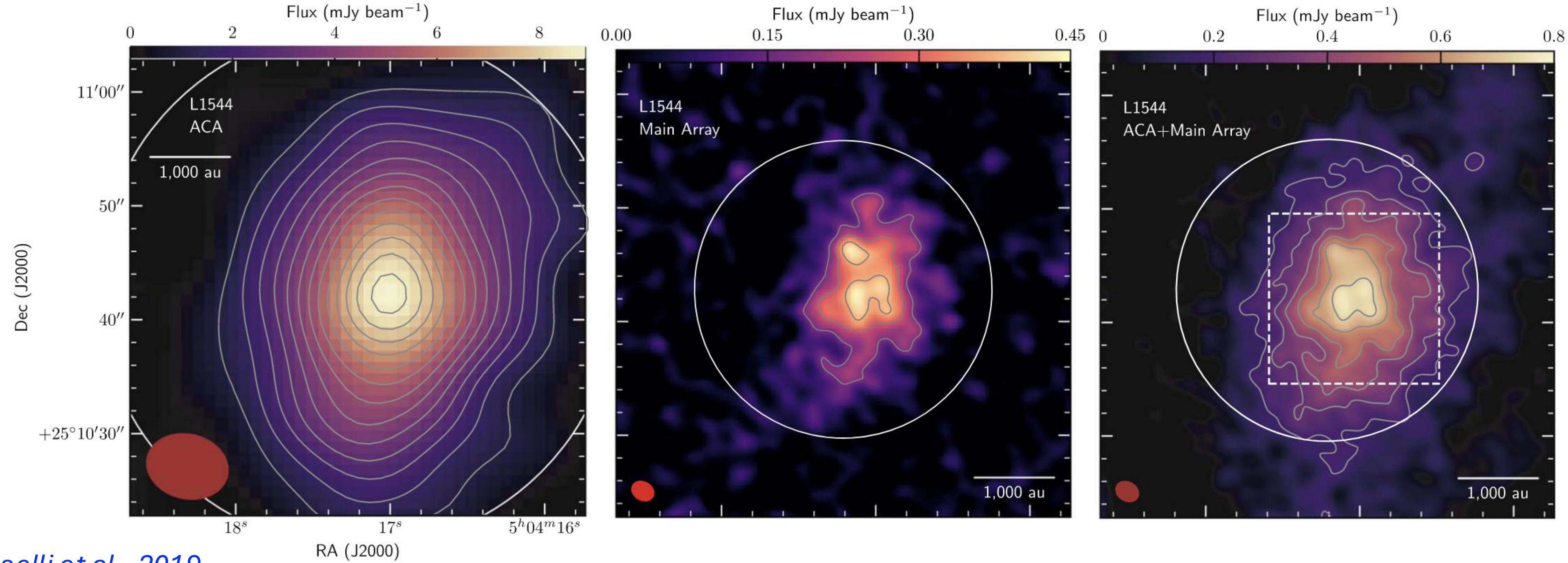
Ideally it would be best to combine with single-dish data, e.g., the GBT!



Credit: ALMA (ESO/NAOJ/NRAO)

"Brightness" Sensitivity

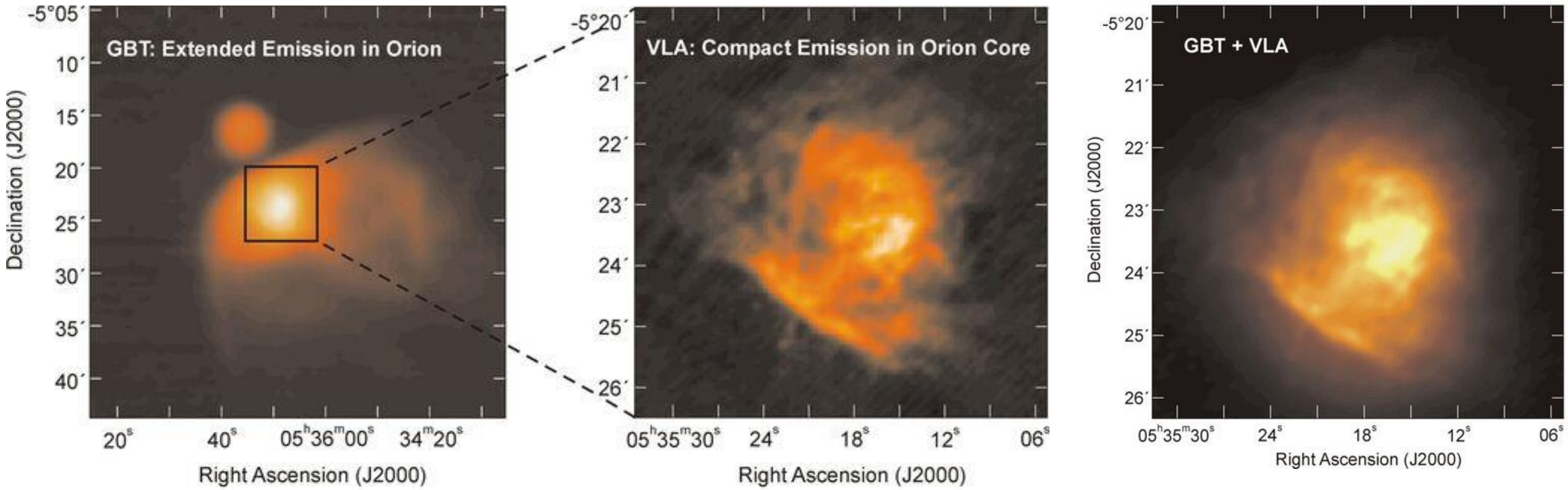
ALMA ACA + 12m image of the Prestellar core L1544



Caselli et al., 2019

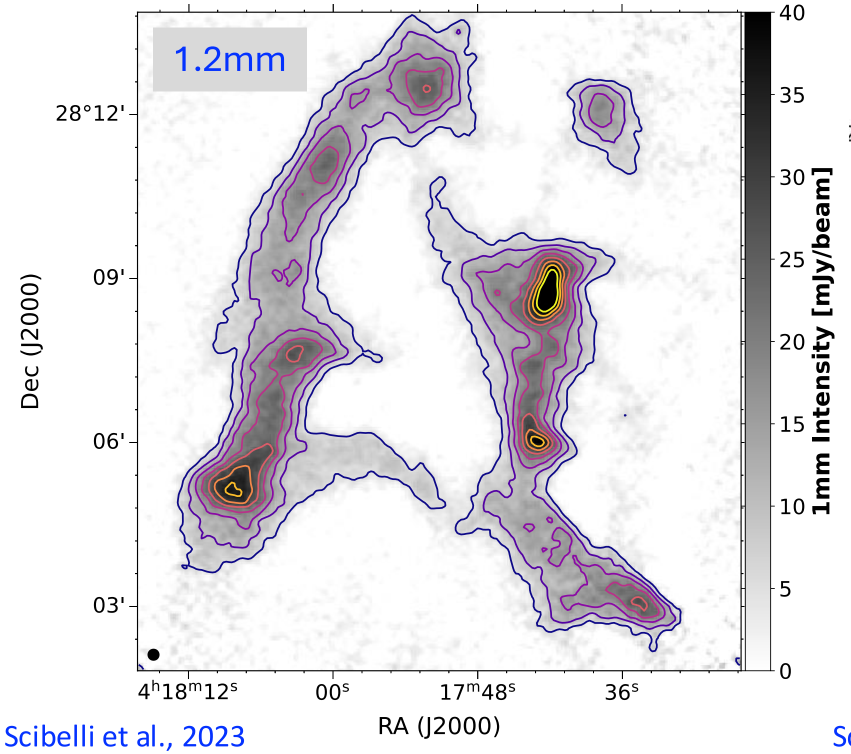
“Brightness” Sensitivity

VLA+GBT image of the Orion Nebula HII region

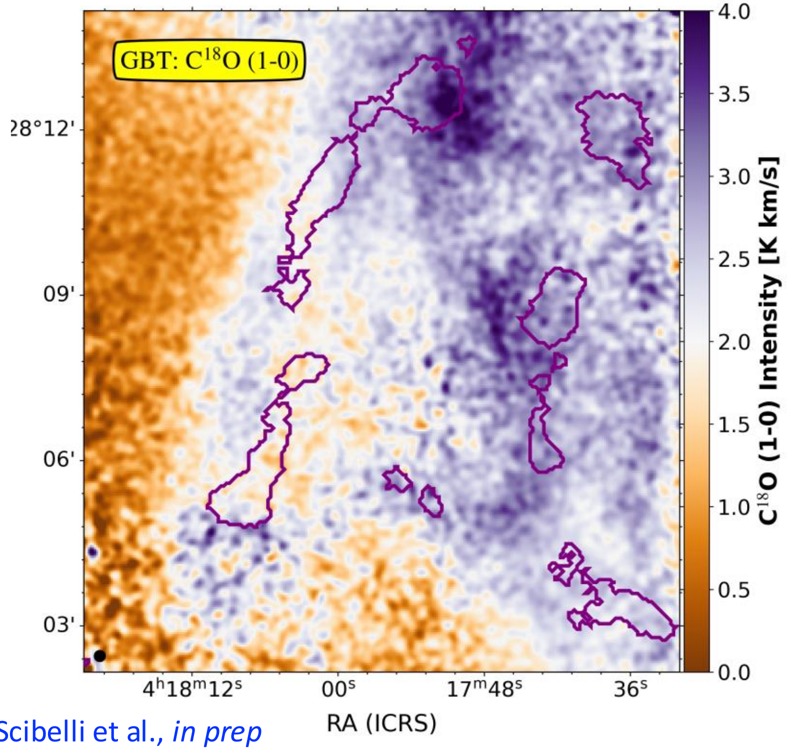


"Brightness" Sensitivity

Often, you'll see maps from any radio telescopes either listed with units of 'Jy/beam' or 'K'
Here I show single dish IRAM 30m and GBT maps in different units, mJy/beam and K km/s, respectively:



Scibelli et al., 2023



Scibelli et al., *in prep*

A proper "spectral brightness" depends only on the source, thus we often use **brightness temperature**, in Kelvin [K]

$$\sigma_T = \left(\frac{\sigma_S}{\Omega_A}\right)\frac{\lambda^2}{2k}. \quad (3.204)$$

https://science.nrao.edu/facilities/vla/proposing/TBconv