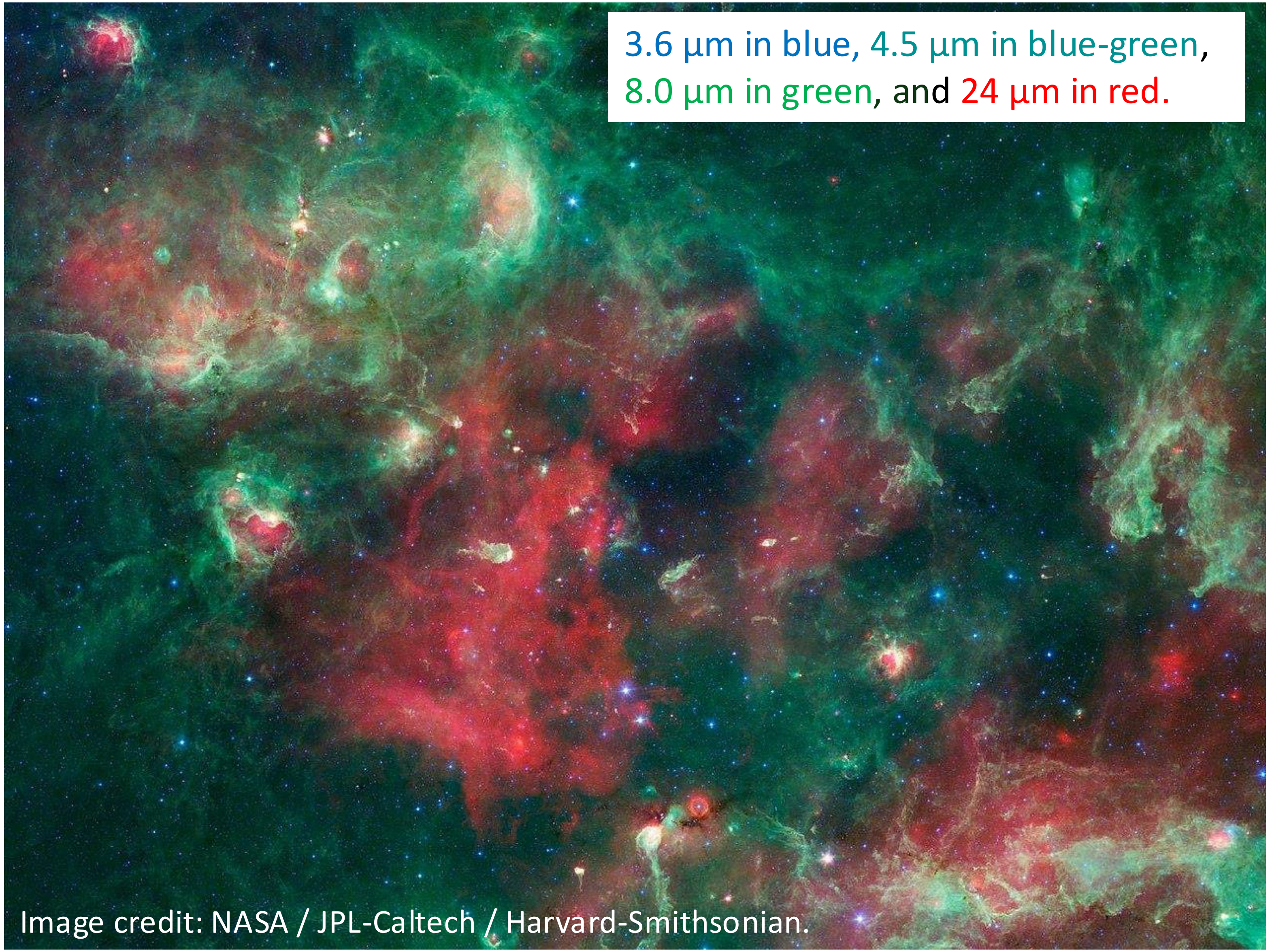
Ionized Gas

HII Regions
Recombination Lines
Free-Free Emission

Cygnus X Star Forming region as imaged by the Spitzer Space Telescope →



Emission Mechanisms

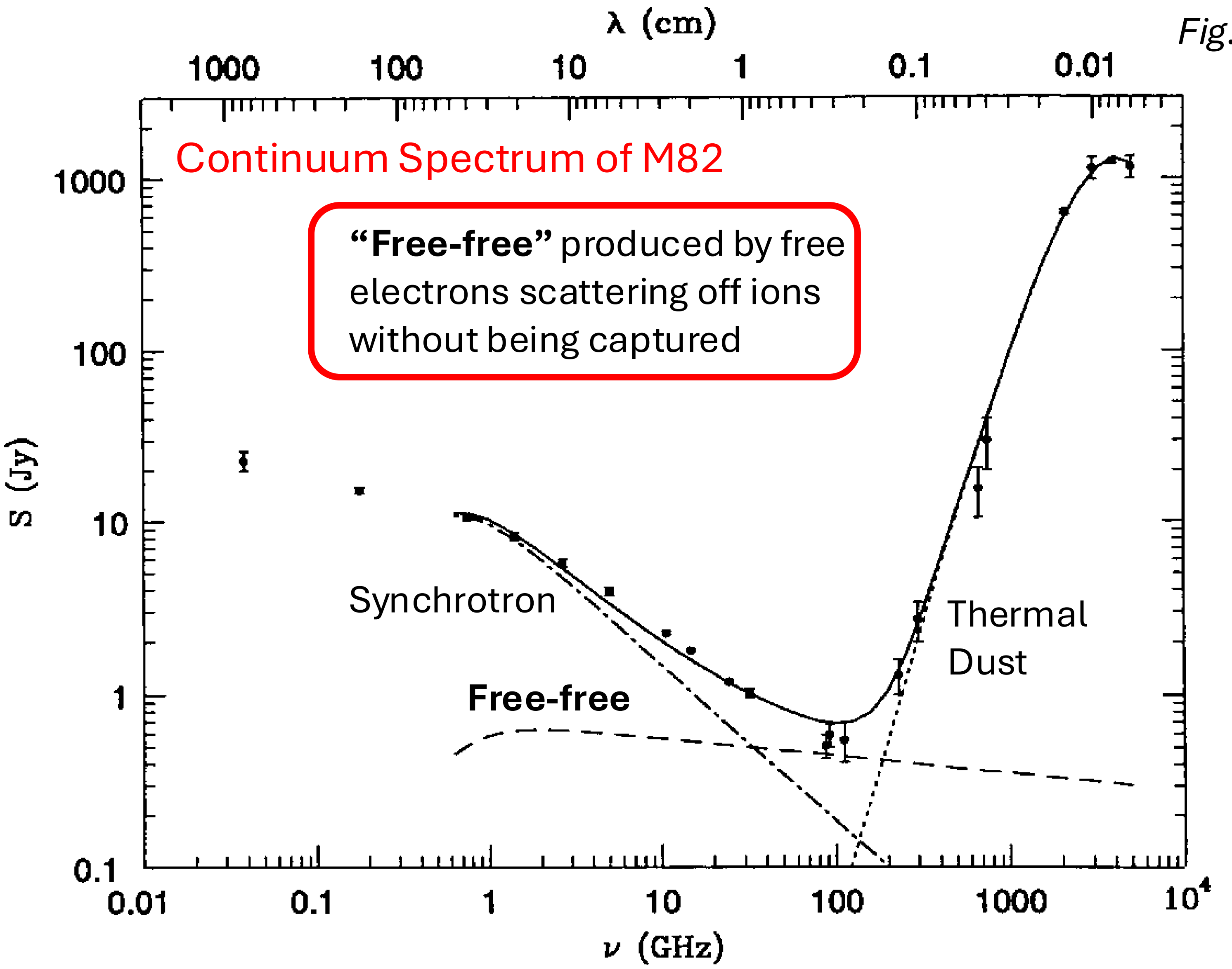
Spectral Lines (ERA Chap. 7)

Free-Free (ERA Chap. 4)

Synchrotron (ERA Chap. 5)

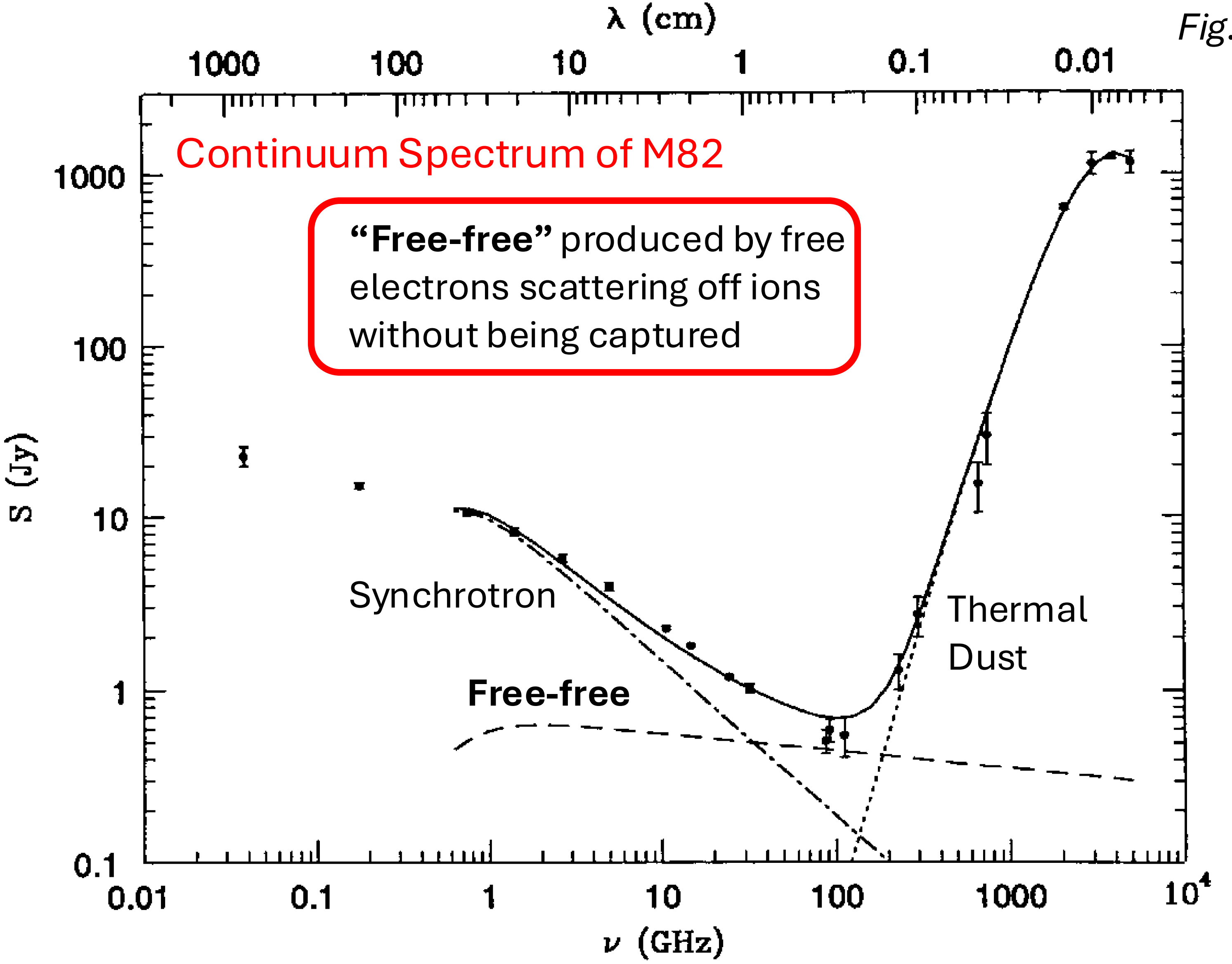
Pulsars (ERA Chap. 6)

Fig. 2.24 (ERA)

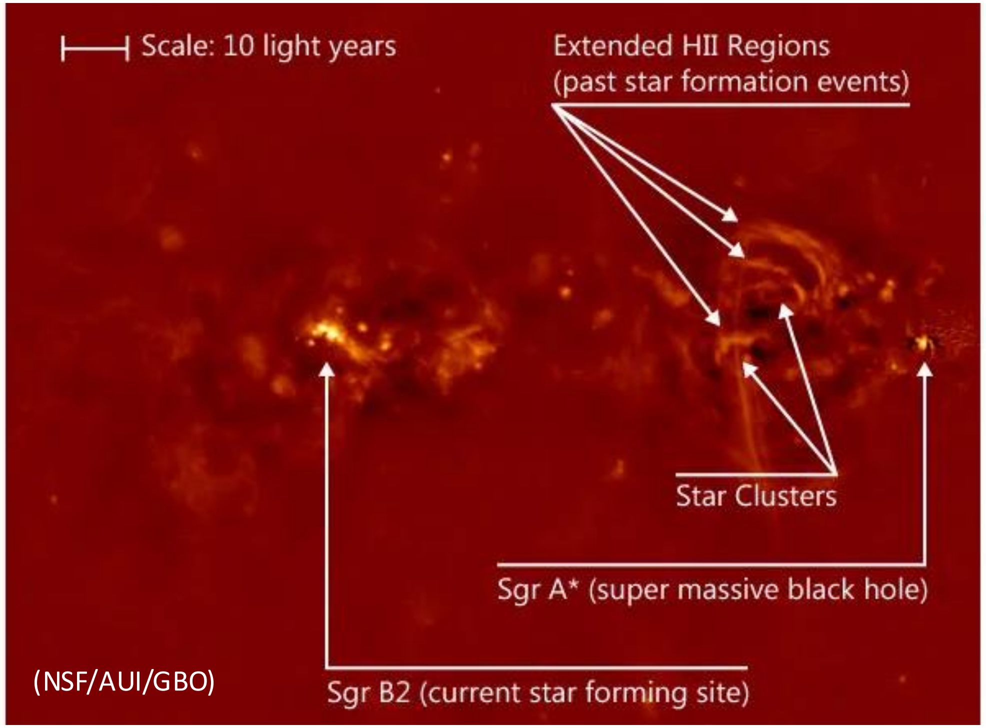


"Free-free" produced by free electrons scattering off ions without being captured

Fig. 2.24 (ERA)



Results from the **Green Bank Observatory** Continuum Instrument, MUSTANG-2, that observes the sky at wavelengths of 3mm (90 GHz)

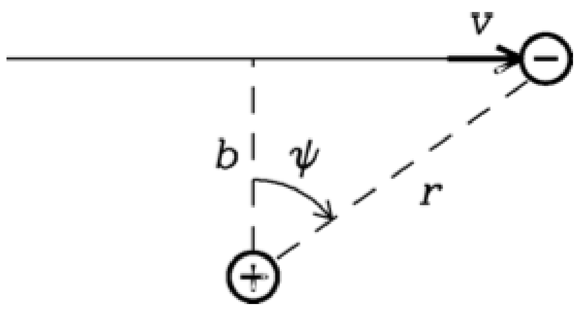


Ginsburg et al., 2020

Free-free Radiation (ERA 4.1)

Thermal and Nonthermal Emission

Free-free: the emission from a charge (e.g., electron) in the Coulomb field of another charge (ion, electron) when it experiences a small deviation in its path



The distance of closest approach, *b*, is called the impact parameter and the interval $\tau = b/v$ is the collision time.

Remember Larmor radiation power is:

$$P = \frac{2q^2\dot{v}^2}{3c^3} \qquad (4.1)$$

More generally, '**bremsstrahlung**' radiation: **electromagnetic radiation** with power P produced by accelerating (or decelerating) an electric charge *q*

NOTE: the magnetic counterpart **magnetobremsstrahlung** or "**magnetic braking radiation**" (e.g., synchrotron radiation) is covered in Chapter 5!

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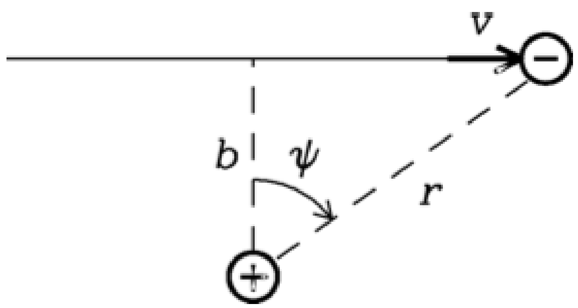

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Typically, '**thermal**' and electrons follow Maxwellian distribution

NOTE: the magnetic counterpart **magnetobremsstrahlung** or "**magnetic braking radiation**" (e.g., synchrotron radiation) is covered in Chapter 5!

Typically, '**nonthermal**' relativistic electrons w/ power-law energy distribution

Free-free Radiation (ERA 4.3)

Free-Free in HII Regions – we need to simply the problem

Main Takeaway

Only electron-ion collisions are important, and only the electrons radiate significantly

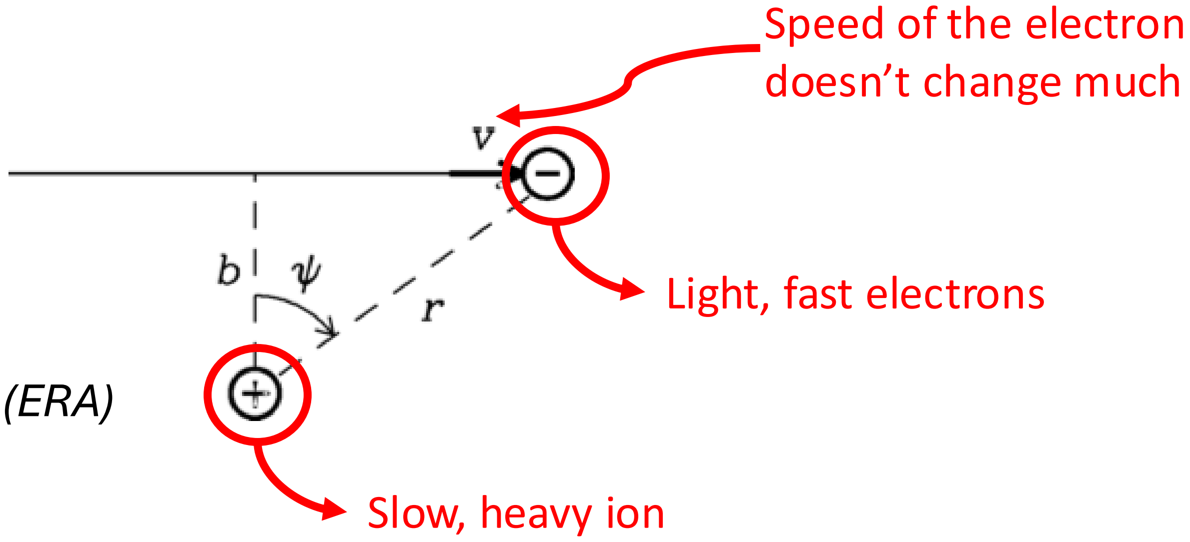


Fig. 4.2 (ERA)

The glowing Trifid Nebula HII region is revealed with near- and mid-infrared views from NASA's Spitzer Space Telescope.

Free-free Radiation (ERA 4.3)

Free-Free in HII Regions – we need to simply the problem

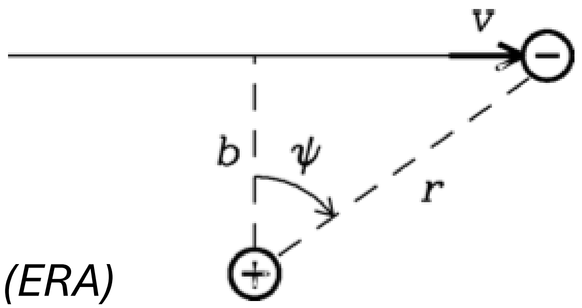


Fig. 4.2 (ERA)

Simplifying the electron-ion scattering problem by estimating the mean **electron energy** in a plasma of temperature *T* and solving for a HII region with T ~ 10^4 K gas,

$$\langle E_e \rangle = \frac{3kT}{2}. \qquad (4.12)$$

$$\langle E_e \rangle \approx \frac{3 \cdot 1.38 \times 10^{-16} \ \text{erg} \ \text{K}^{-1} \cdot 10^4 \ \text{K}}{2} \approx 2 \times 10^{-12} \ \text{erg} \approx 1 \ \text{eV}. \qquad (4.13)$$

Compared to the **energy of a radio photon only**,

$$E = h\nu \approx 6.63 \times 10^{-27} \ \text{erg s} \cdot 10^{10} \ \text{Hz} \approx 6.63 \times 10^{-17} \ \text{erg} \approx 4 \times 10^{-5} \ \text{eV}. \qquad (4.14)$$

Lost energy is so small compared to the initial energy of the electron

Main Takeaway

OK to approximate the path of the electron as a straight line/linear interaction

Free-free Radiation (ERA 4.3)

Free-Free in HII Regions – we need to simply the problem

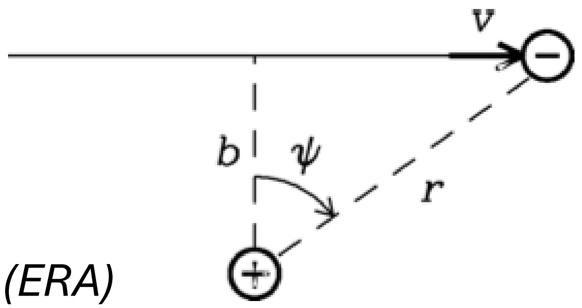


Fig. 4.2 (ERA)

While the velocity is being approximated as constant, doesn't mean it is not feeling the acceleration, which is the important part to understand this radiation!

During the interaction the electron will be accelerated electrostatically both parallel and perpendicular to its nearly straight path,

$$F_{\parallel} = m_e \dot{v}_{\parallel} = \frac{-Ze^2}{r^2} \sin\psi = \frac{-Ze^2 \sin\psi \cos^2\psi}{b^2}, \qquad (4.15)$$

$$F_{\perp} = m_e \dot{v}_{\perp} = \frac{Ze^2}{r^2} \cos\psi = \frac{Ze^2 \cos^3\psi}{b^2}, \qquad (4.15)$$

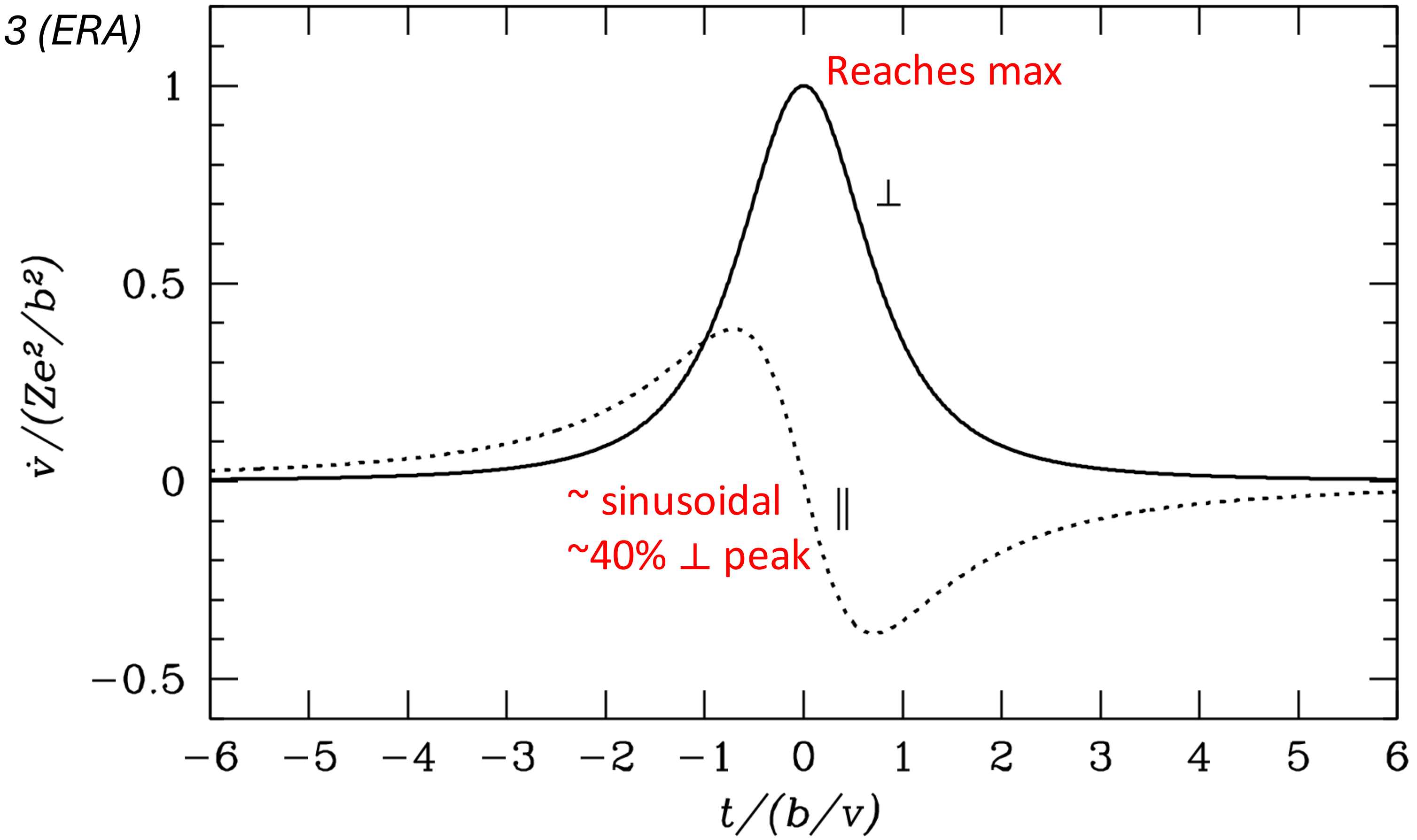
Written in terms of impact parameter 'b' where remember $\tau = b/v$ is the collision time.

Let's look at this graphically…

Free-free Radiation (ERA 4.3)

Free-Free in HII Regions – we need to simply the problem

Fig. 4.3 (ERA)



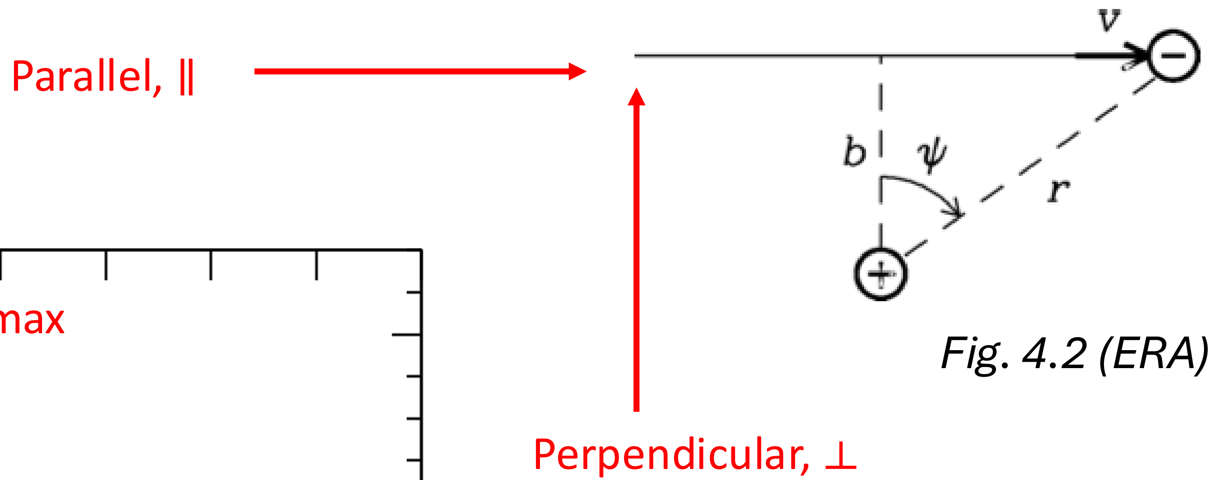
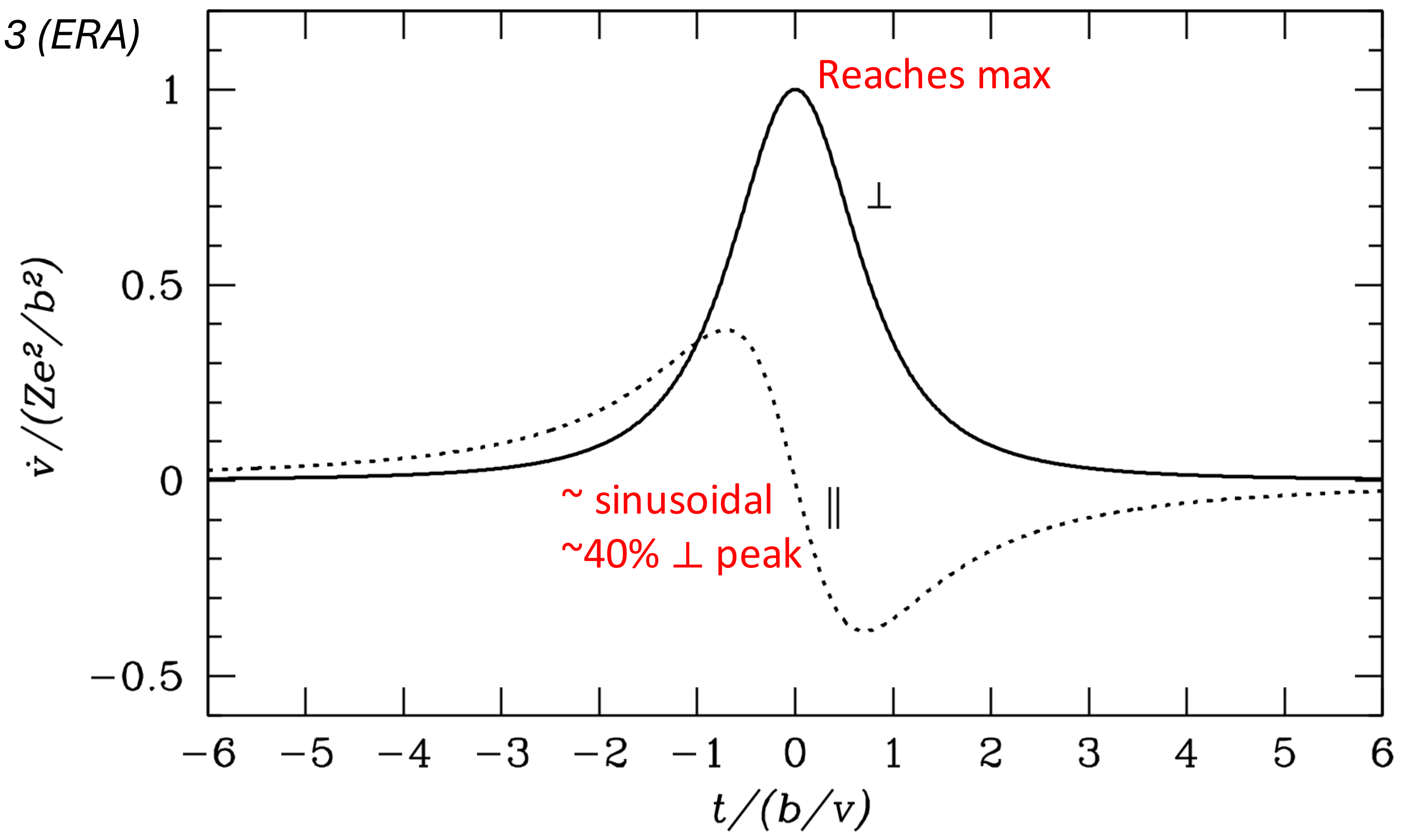


Fig. 4.2 (ERA)

Free-free Radiation (ERA 4.3)

Free-Free in HII Regions – we need to simply the problem

Fig. 4.3 (ERA)



Parallel, ∥

Perpendicular, ⊥

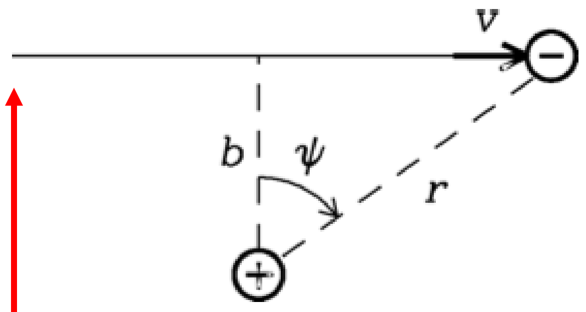


Fig. 4.2 (ERA)

Main Takeaway

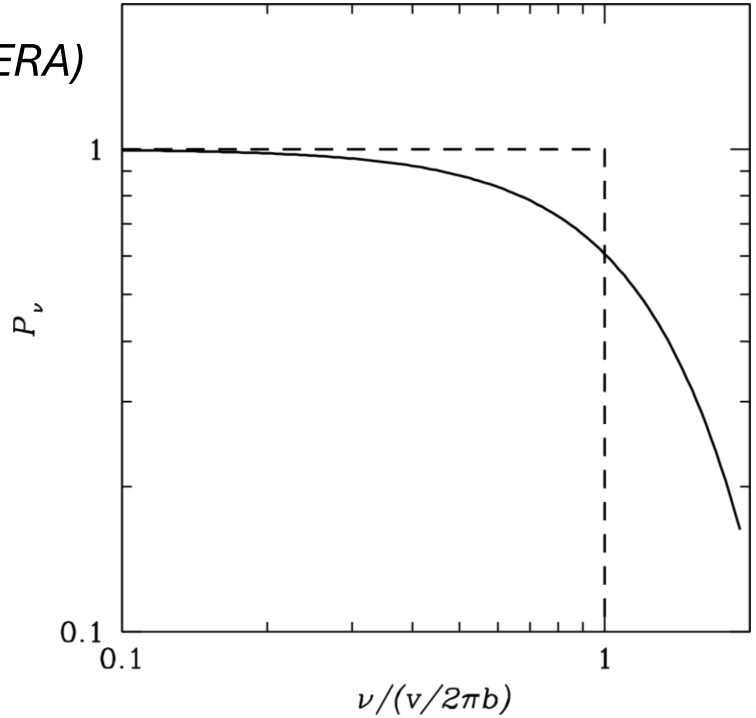
Periodic parallel acceleration gives signal very weak at radio frequencies.

We only need to worry about the perpendicular motion in the radio!

Free-free Radiation (ERA 4.3)

Free-Free in HII Regions – we need to simply the problem

Fig. 4.4 (ERA)



So, we take the form of the **perpendicular component and plug into Larmor's formula** and **integrate over all frequencies and final velocities to get our power spectrum** (see textbook for derivation details!):

$$P = \frac{2}{3}\frac{e^2 \dot{v}_\perp^2}{c^3} = \frac{2e^2}{3c^3}\frac{Z^2 e^4}{m_e^2}\left(\frac{\cos^3\psi}{b^2}\right)^2. \qquad (4.17)$$

$$W = \int_{-\infty}^{\infty} P\,dt. \qquad (4.18)$$

$$W = \frac{\pi Z^2 e^6}{4c^3 m_e^2}\left(\frac{1}{b^3 v}\right). \qquad (4.24)$$

From converting dt to dψ and solving for ***W*, which is the pulse energy radiated by a single electron-ion interaction** characterized by impact parameter *b* and velocity *v*.

Free-free Radiation (ERA 4.3)

Free-Free in HII Regions – we need to simply the problem

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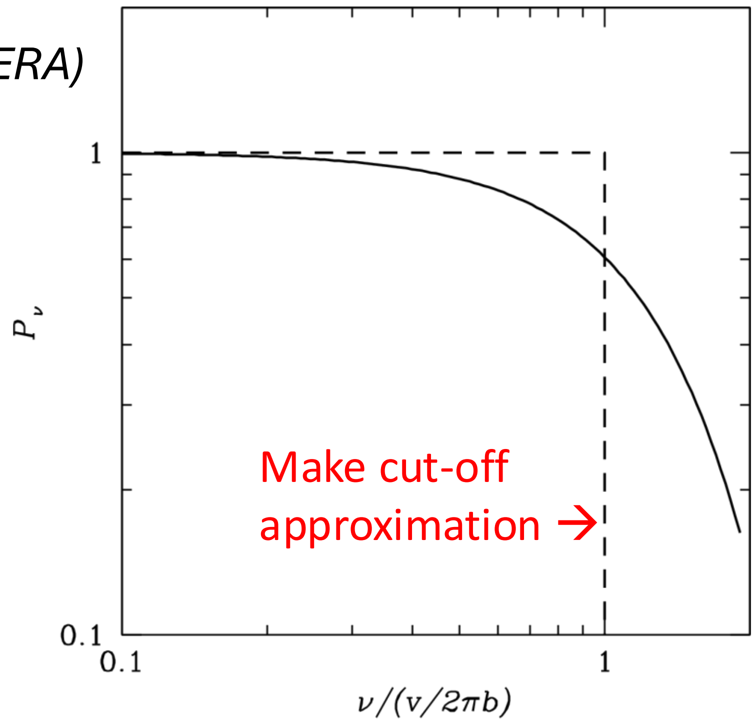
From converting dt to dψ and solving for ***W*, which is the pulse energy radiated by a <u>single electron-ion interaction</u>** characterized by impact parameter *b* and velocity *v*.

In the flat approximation the average energy per unit frequency becomes,

$$W_\nu \approx \frac{W}{\nu_{\max}} = \left(\frac{\pi Z^2 e^6}{4c^3 m_e^2 b^3 v}\right)\left(\frac{2\pi b}{v}\right), \quad (4.25)$$

Fig. 4.4 (ERA)



Main Takeaway

This energy is emitted in a single pulse of duration $\tau \approx b/v$, so the pulse **power spectrum** is nearly **flat over all frequencies** $\nu < \nu_{max} \approx (2\pi\tau)-1 \approx v/(2\pi b)$ and **falls rapidly at higher frequencies** that fall outside the radio: **$\nu < \nu_{max} \sim 10^{14}$ Hz**

Free-free Radiation (ERA 4.3)

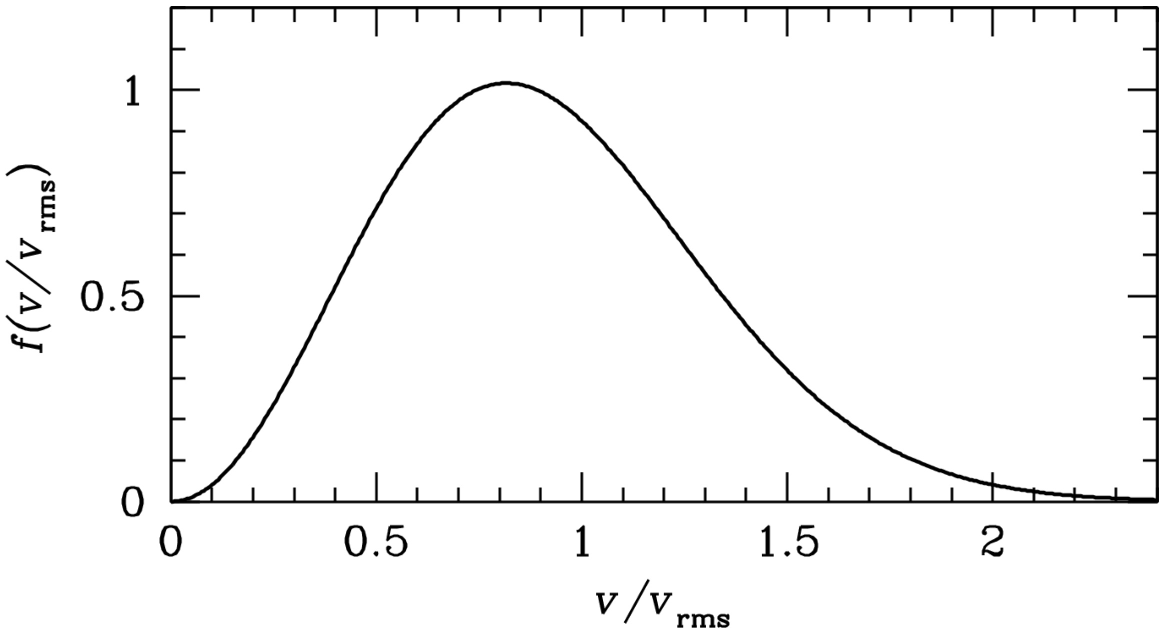
Radio Radiation from an HII Region (LTE approximation)

The strength and spectrum of radio emission from an HII region depends on the distributions of electron velocities *v* and collision impact parameters *b*, that follow a **Maxwellian speed distribution that depends on mass (m_e) and temperature (T).**

$$f(v) = \frac{4v^2}{\sqrt{\pi}} \left(\frac{m_e}{2kT} \right)^{3/2} \exp\left(-\frac{m_e v^2}{2kT} \right). \qquad (4.34)$$

(see appendix B.8 for derivation)

Fig. 4.6 (ERA)



GREEN BANK OBSERVATORY
National Radio Astronomy Observatory
NRAO
NSF

Free-free Radiation (ERA 4.3)

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See book for details on derivation(!), but once we **integrate over all impact parameters and velocities** we can write in our **free-free emission coefficient** as,

$$j_\nu = \frac{\pi^2 Z^2 e^6 n_e n_i}{4c^3 m_e^2} \left(\frac{2m_e}{\pi k T} \right)^{1/2} \ln \left(\frac{b_{max}}{b_{min}} \right). \qquad (4.39)$$

Where b_{min} and b_{max} are the minimum and maximum impact parameters (NOTE: small uncertainties in these values have very little effect on the calculated emission coefficient in an HII region).

Free-free Radiation (ERA 4.3)

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Where b_{min} and b_{max} are the minimum and maximum impact parameters (NOTE: small uncertainties in these values have very little effect on the calculated emission coefficient in an HII region).

We know in **LTE we can use Kirchhoff's law** to find the **absorption coefficient** and the blackbody brightness,

$$\kappa = \frac{j_\nu}{B_\nu(T)} \approx \frac{j_\nu c^2}{2kT\nu^2} \quad (4.51)$$

Which, in the **Rayleigh-Jeans limit** becomes,

$$\kappa = \frac{1}{\nu^2 T^{3/2}} \left[\frac{Z^2 e^6}{c} n_e n_i \frac{1}{\sqrt{2\pi (m_e k)^3}}\right] \frac{\pi^2}{4} \ln\left(\frac{b_{max}}{b_{min}}\right). \quad (4.52)$$

Main Takeaway

$\kappa \propto \nu^{-2.1}$

Free-free Radiation (ERA 4.3)

Radio Radiation from an HII Region (LTE approximation)

As we've done before, we can integrate our absorption coefficient along the line of sight to get an **optical depth!**

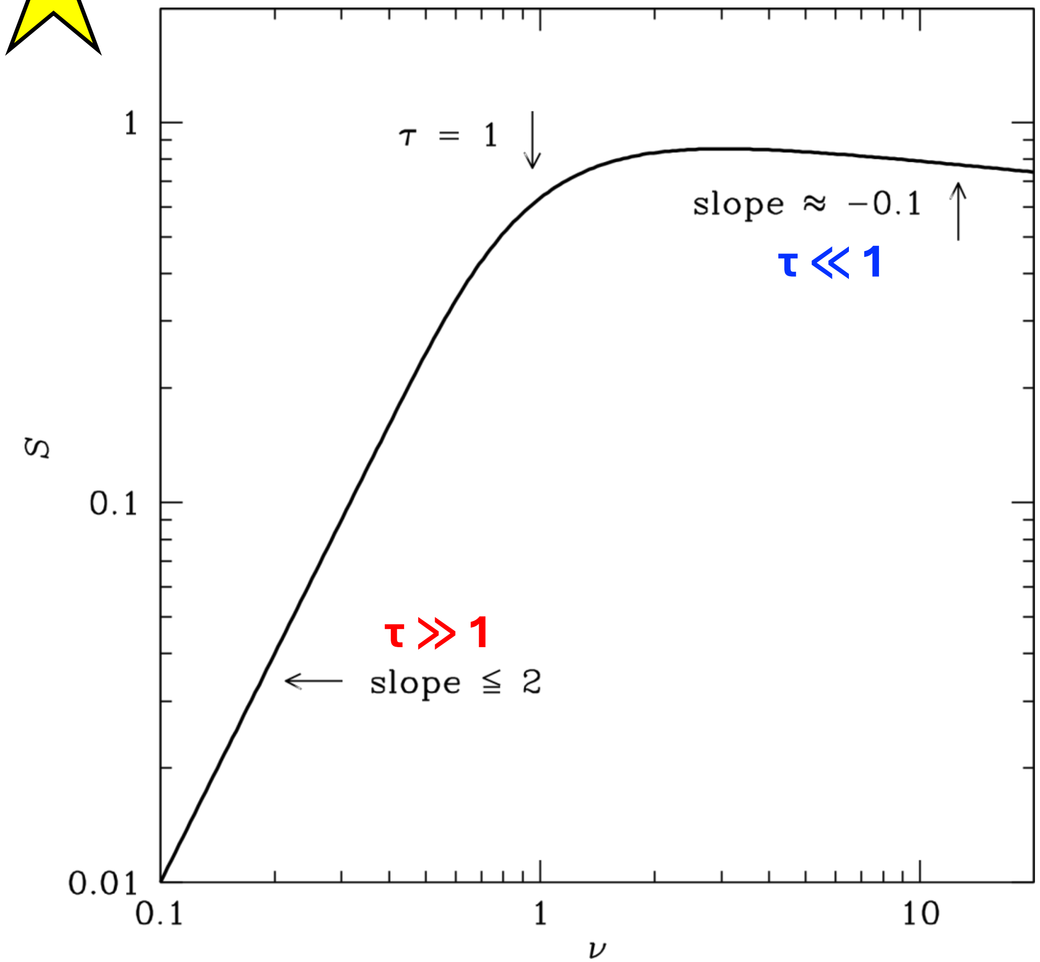
$$\tau = -\int_{\rm los} \kappa \, ds \propto \int \frac{n_e n_i}{\nu^{2.1} T^{3/2}} ds \approx \int \frac{n_e^2}{\nu^{2.1} T^{3/2}} ds. \qquad (4.53)$$

At frequencies low enough that **$\tau \gg 1$**, the HII region becomes opaque, its spectrum approaches that of a blackbody with brightness temperature approaching the electron temperature ($T_b \approx T \sim 10^4$ K), and its flux density obeys the Rayleigh–Jeans approximation **$S \propto \nu^2$**. At very high frequencies, **$\tau \ll 1$**, the HII region is nearly transparent, and

$$S \propto \frac{2kT\nu^2}{c^2} \tau(\nu) \propto \nu^{-0.1}. \qquad (4.54)$$

Fig. 4.8 (ERA)

The overall radio spectrum of a uniform HII region



Free-free Radiation (ERA 4.3)

Radio Radiation from an HII Region (LTE approximation)

The spectral slope on a log-log plot is often called the **spectral index**

$$\alpha \equiv \pm \frac{d \log S}{d \log \nu}. \qquad (4.55)$$

BEWARE! both sign conventions are found in the literature

With the + sign convention, **the low-frequency spectral index of a uniform HII region would be α = +2 and for an inhomogeneous HII region will be α ≈− 0.1**

Fig. 4.8 (ERA)

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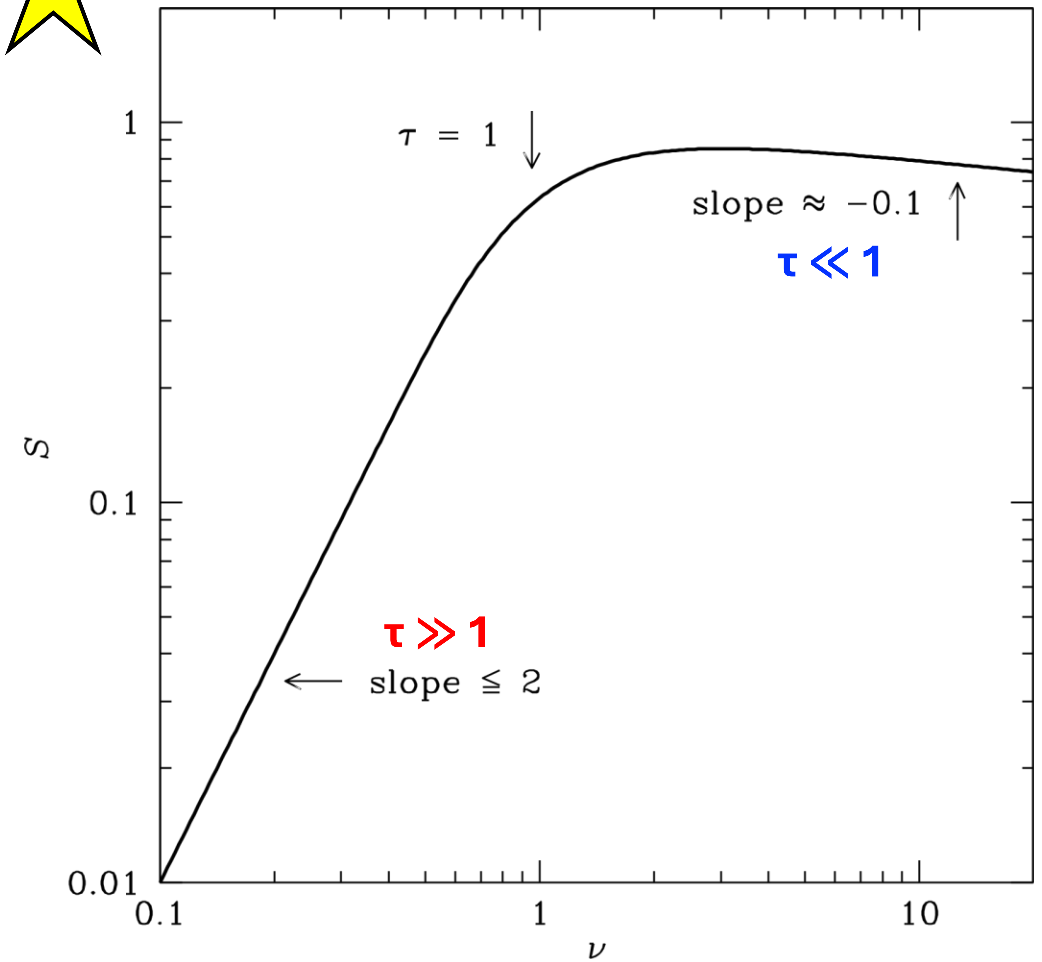


Free-free Radiation (ERA 4.3)

Radio Radiation from an HII Region (LTE approximation)

Remember from recombination lines that we define an **emission measure**,

$$\frac{\text{EM}}{\text{pc}\ \ \text{cm}^{-6}} \equiv \int_{\text{los}} \left(\frac{n_e}{\text{cm}^{-3}}\right)^2 d\left(\frac{s}{\text{pc}}\right). \qquad (4.57)$$

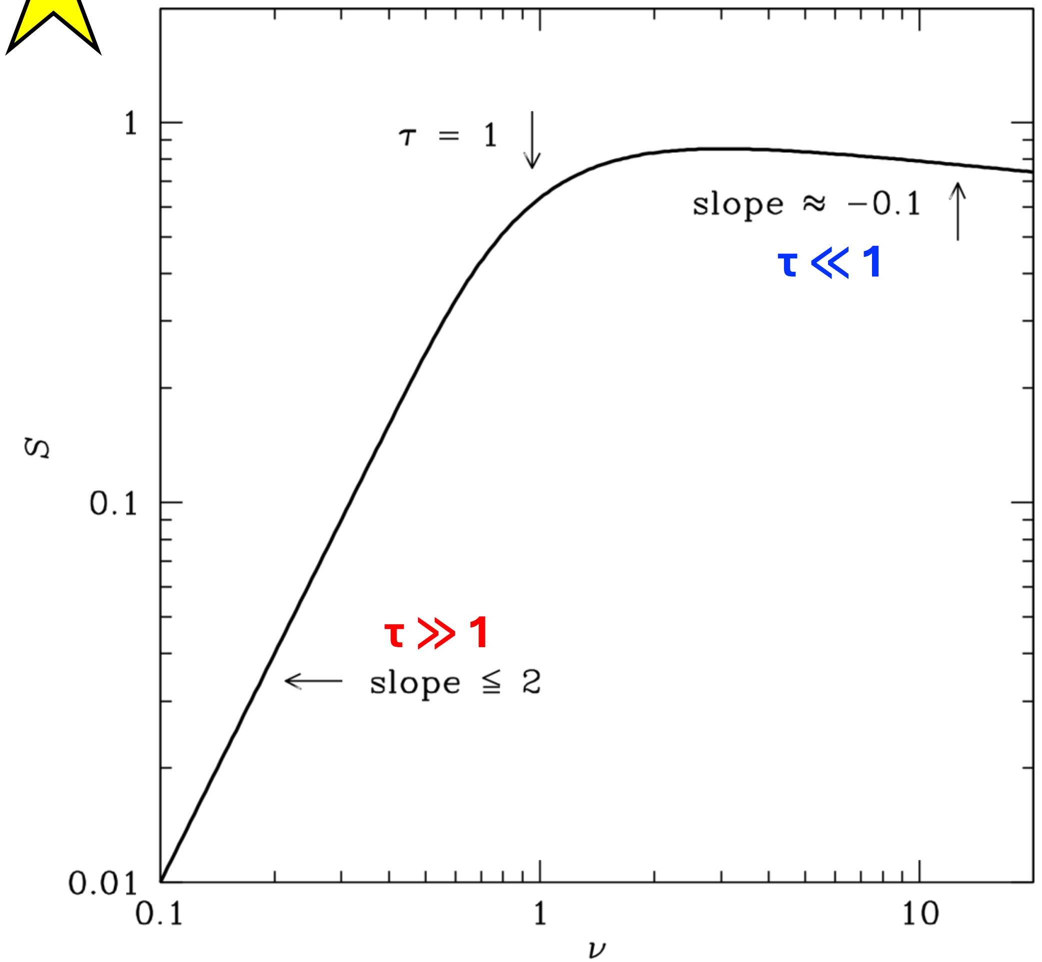
That is related to our optical depth:

$$\tau \approx 3.28 \times 10^{-7} \left(\frac{T}{10^4\ \text{K}}\right)^{-1.35} \left(\frac{\nu}{\text{GHz}}\right)^{-2.1} \left(\frac{\text{EM}}{\text{pc}\ \ \text{cm}^{-6}}\right). \qquad (4.60)$$

Wooohooo! We can get estimates for electron temperature, electron density, emission measure, and production rate of ionizing photons

Fig. 4.8 (ERA)

The overall radio spectrum of a uniform HII region



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The electron temperature, T, from the brightness temperature:

$$T_b = T\left(1 - e^{-\tau}\right) \qquad (4.61)$$

Production rate of ionizing photons:

$$\left(\frac{Q_H}{\mathrm{s^{-1}}}\right) \approx 6.3 \times 10^{52} \left(\frac{T}{10^4\ \mathrm{K}}\right)^{-0.45} \left(\frac{\nu}{\mathrm{GHz}}\right)^{0.1} \left(\frac{L_\nu}{10^{20}\ \mathrm{W\ \ Hz^{-1}}}\right). \qquad (4.62)$$

See book for some examples

Free-free Radiation (ERA 4.3)

Radio Radiation from an HII Region (LTE approximation)

Important practical application for starburst galaxies, luminous infrared galaxies (LIRGs), etc.,

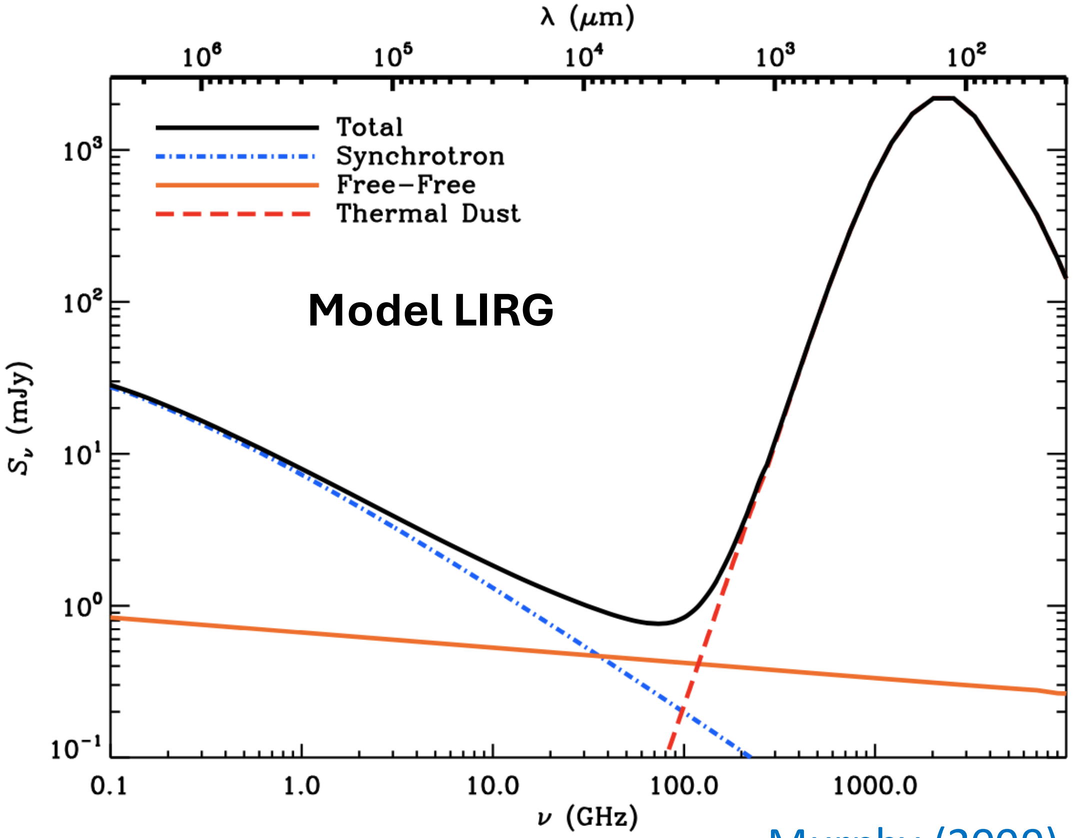
If the free-free and synchrotron emission are roughly cospatial, its radio brightness temperature at frequencies ν<100 GHz is,

$$T_b \sim T\,[1 - \exp(-\tau)]\left[1 + 10\left(\frac{\nu}{\mathrm{GHz}}\right)^{0.1+\alpha}\right], \qquad (4.63)$$

where $T \approx 10^4$ K and α -0.8 is the spectral index of the synchrotron radiation.

Free–free absorption of the synchrotron radiation limits the maximum brightness temperature to $T_b \leq 10^5$ K at frequencies $\nu \geq 1$ GHz.

This limit can be used to identify the energy source powering a compact radio source at the center of a galaxy: **if its brightness temperature is significantly higher than 10^5 K, it is powered by an AGN, not a compact starburst.**



Murphy (2009)

Free-free Radiation (ERA 4.3)

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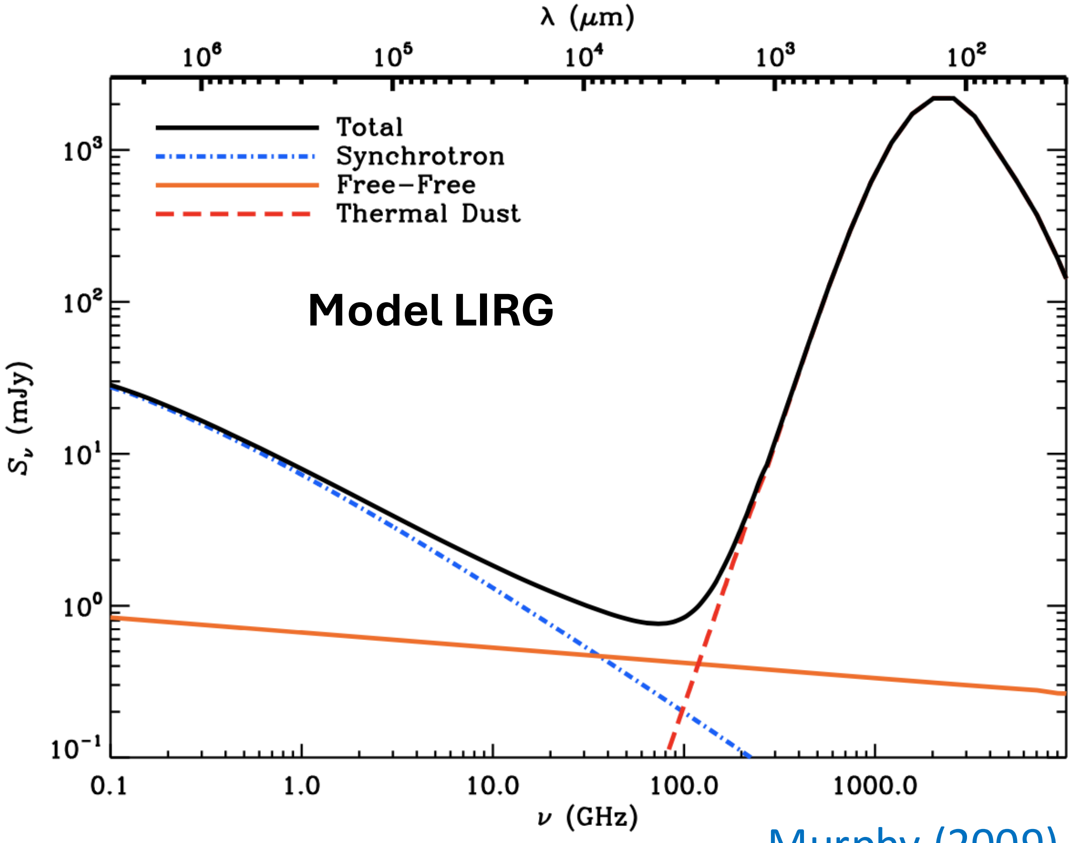

Murphy (2009)

Main Takeaway

Disentangling the "thermal" component from the "non-thermal" one is difficult below 30 GHz!

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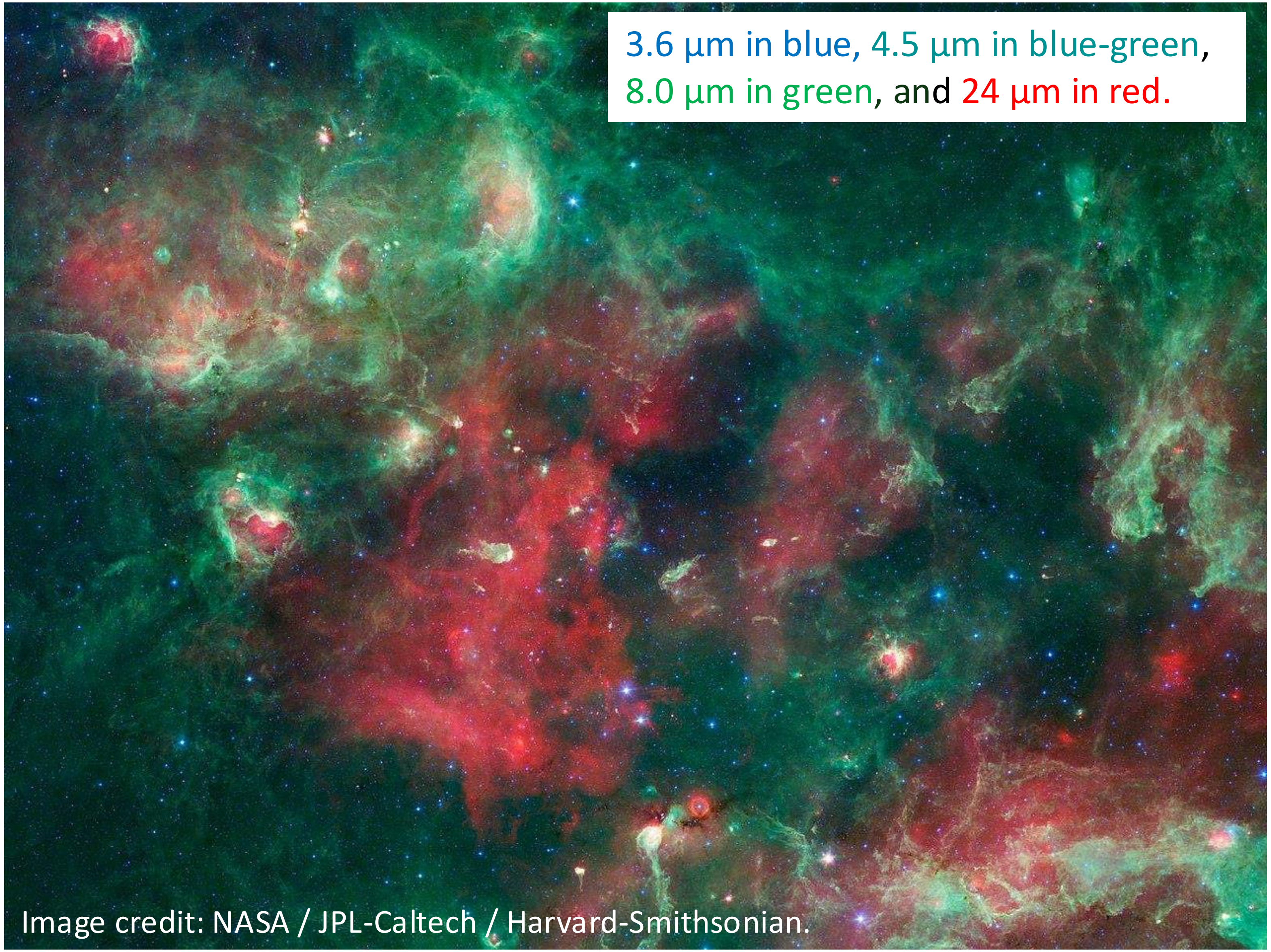
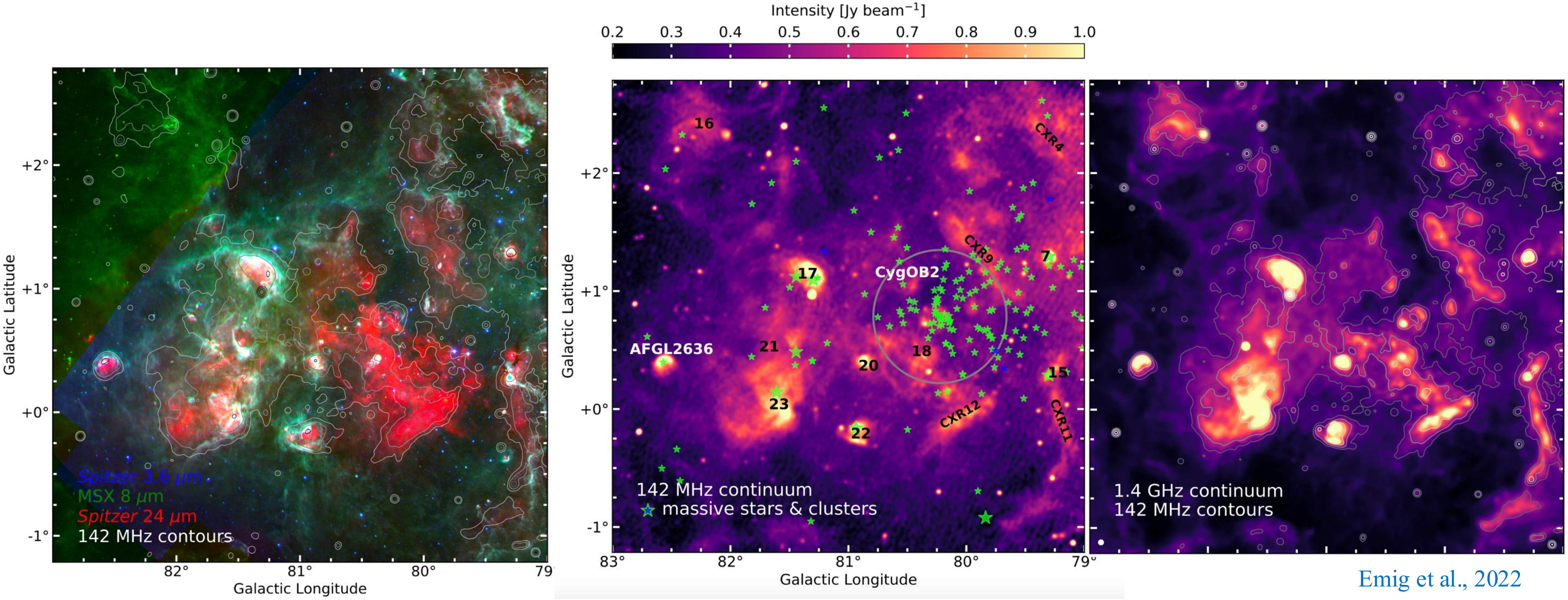
Free-free Observations

Remember the Cygnus X region?



Free-free Observations

The Galactic radio emission in the region analyzed is almost **entirely thermal (free-free) at 148 MHz**, with emission measures (EM) of $10^3 < \mathrm{EM}\ [\mathrm{pc\ cm^{-6}}] < 10^6$



Emig et al., 2022

Free-free Observations

Created a short-spacing map at 142 MHz and with 72' resolution that is interpolated from multi-frequency fits across 52 MHz, 408 MHz, 820 MHz, and 1420 MHz!

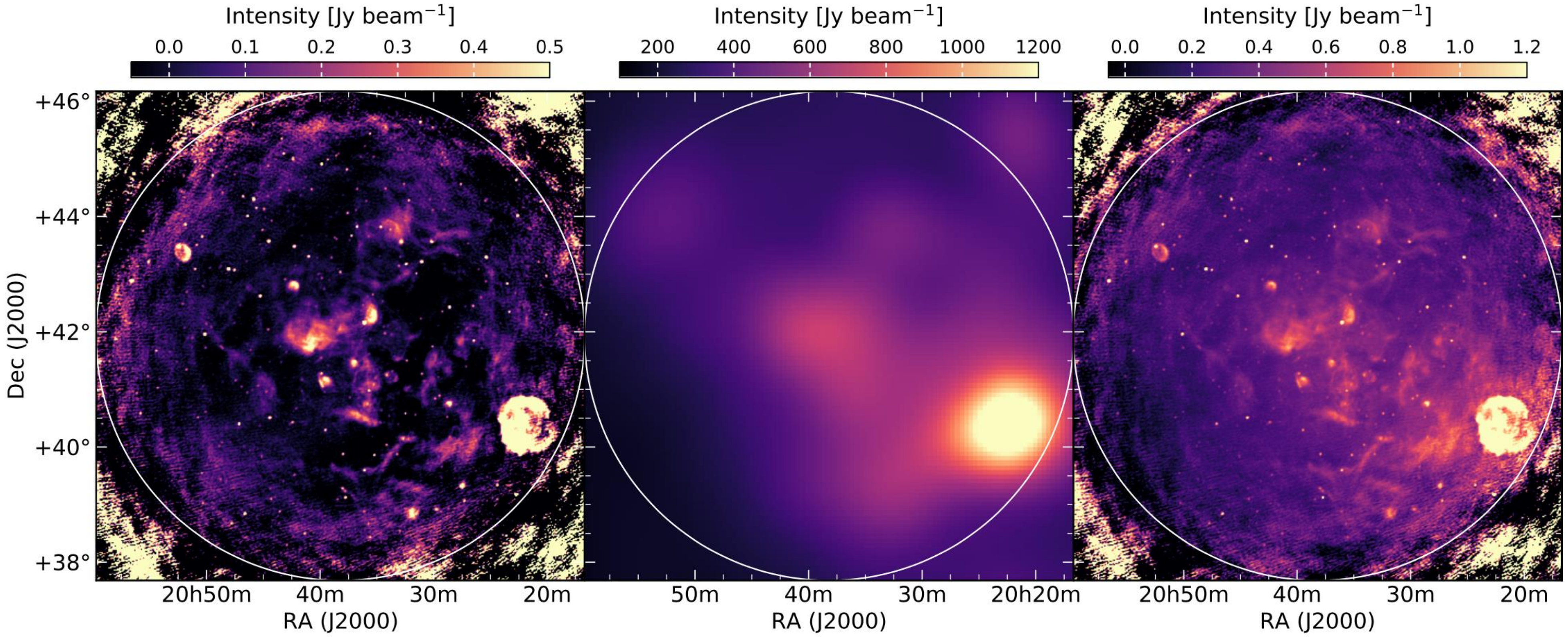


Figure 2.2: Imaging LOFAR 142 MHz observations of the Cygnus X region. The white circle shows the 20% power of the primary beam. We draw attention to the different intensity scales among the images. For reference, Cygnus A is located about 7.4°due west from the center of the pointing. *Left:* Deconvolved LOFAR image without short-spacing information. While these data are sensitive to emission on angular scales as large as 96′, negative (un-physical) emission is present induced by large scale emission in the Galactic plane. *Center:* The frequency-interpolated short-spacing map at 142 MHz (see Section 2.2.1) and 72′ resolution. *Right:* LOFAR data imaged with the short-spacing map as an initial model (see Section 2.2.1).

Emig et al., 2022

Free-free Observations

Did this correction for short spacing to interpolate at 142 MHz by using this correction for free-free and synchrotron emission!

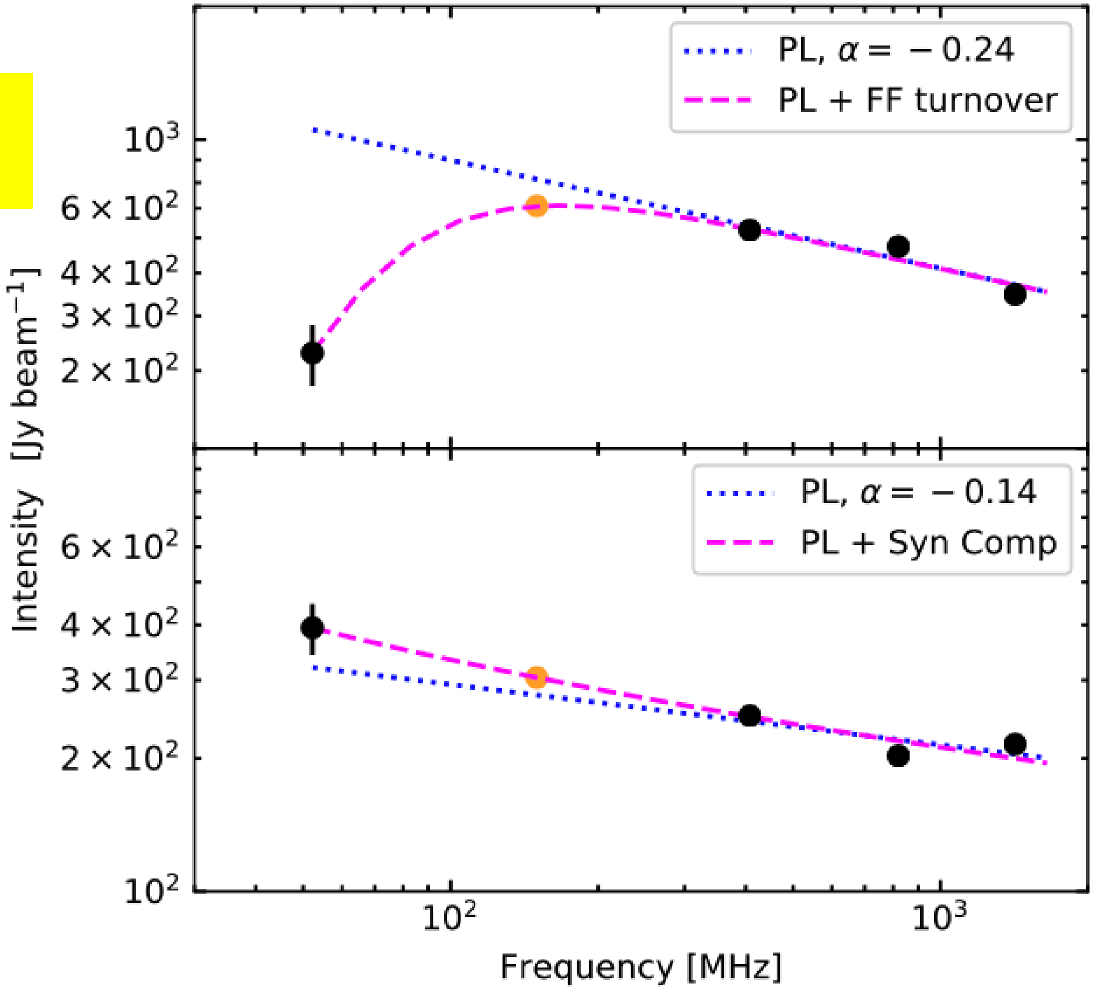
"Pixel by pixel, we fit a power-law to the "high-frequency" data points at 408, 820 and 1420 MHz, as previous studies have found this region largely consists of thermal, free-free emission down to 408 MHz (Landecker 1984; Xu et al. 2013). Then, to the 52 MHz data point, we fit for either a free-free turnover or a synchrotron component." →

Just free-free:

$$S(\nu) = \frac{S_{0,\mathrm{ff}}}{\tau_0}\left(\frac{\nu}{\nu_0}\right)^2\left(1-\exp\left[-\tau_0\left(\frac{\nu}{\nu_0}\right)^{\alpha-2}\right]\right)$$

w/ synchrotron correction:

$$S(\nu) = S_{0,\mathrm{ff}}\left(\frac{\nu}{\nu_0}\right)^{\alpha} + S_{0,\mathrm{s}}\left(\frac{\nu}{\nu_0}\right)^{-0.7}$$

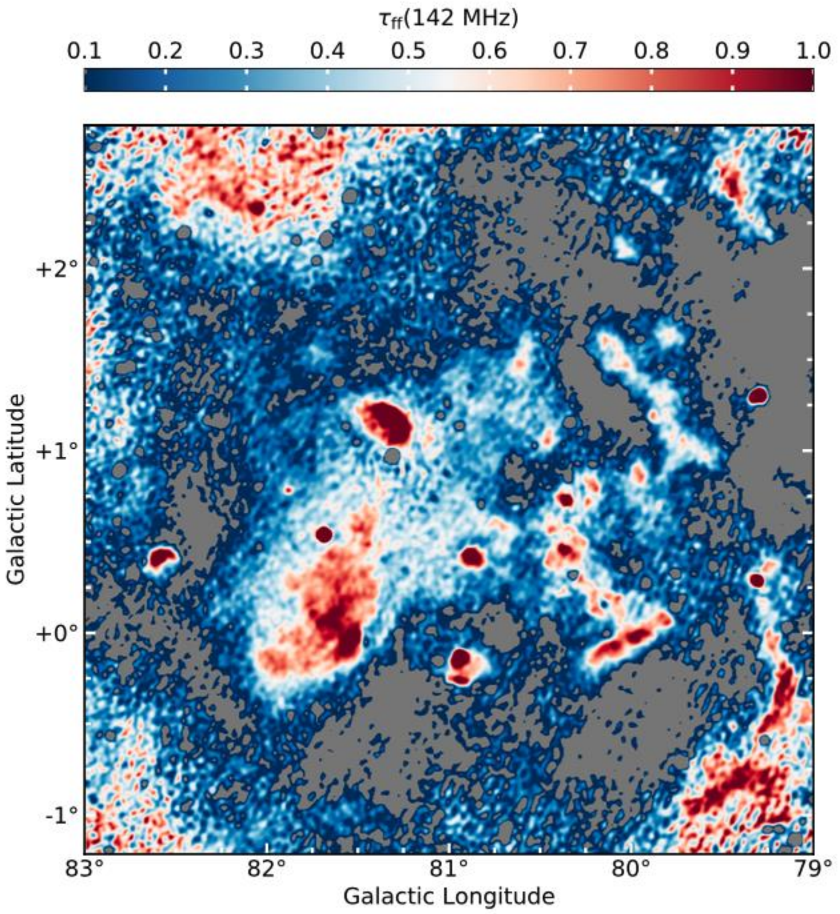


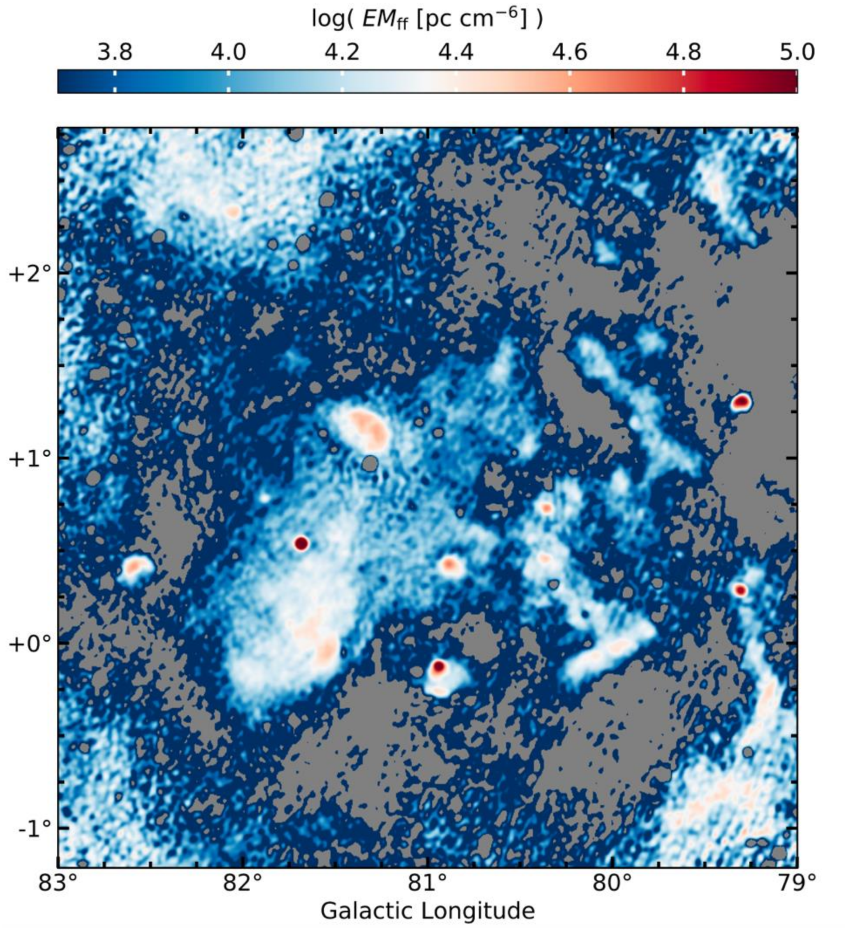
Emig et al., 2022

Free-free Observations

optical depth, $\tau_{\rm ff}$, is given by (Condon 1992; Emig et al. 2020b),

$$\tau_{\rm ff}(\nu) = 3.37 \times 10^{-4} \left(\frac{EM_+}{10^3\ {\rm cm}^{-6}\ {\rm pc}} \right) \left(\frac{T_e}{10^4\ {\rm K}} \right)^{-1.323} \left(\frac{\nu}{1\ {\rm GHz}} \right)^{-2.118}$$





Same equation 4.60 in ERA! 

From estimated optical depth in SED fit, and assuming T_e ~ 7,500 K *(estimated from previous recombination line measurements!)* they map the EM!

Emig et al., 2022

Emission Mechanisms

Spectral Lines (ERA Chap. 7)

Free-Free (ERA Chap. 4)

Synchrotron (ERA Chap. 5)

Pulsars (ERA Chap. 6)

Synchrotron Radiation (ERA Chapter 5)

Magnetobremsstrahlung (5.1)

Remember: the lightest particles are accelerated more so **electrons account for virtually all of the radiation observed**

Types:

- **Gyro radiation**: electrons with velocities much less than the speed of light $v \ll c$
- **Cyclotron radiation:** mildly relativistic electrons w/ kinetic energy comparable to the rest mass $m_e c^2$
- **Synchrotron radiation:** ultra relativistic electrons with kinetic energy $\gg$ rest mass $m_e c^2$

Synchrotron Radiation (ERA Chapter 5)

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- Accounts for **most of the radio emission from** 'Active Galactic Nuclei' or **AGNs** that are thought to be powered by supermassive black holes in galaxies and quasars
- **Dominates radio continuum emission of star forming galaxies at ν < 30 GHz**
- Magnetosphere of Jupiter is also a synchrotron source
- Synchrotron radiation also presents as...
  - optical emission from Crab Nebula
  - optical jet of M87
  - optical → X-ray emission of quasars

Synchrotron Radiation (ERA Chapter 5)

Magnetobremsstrahlung (5.1)

(5.1.1) **Gyro radiation**: electrons with velocities much less than the speed of light $v \ll c$

Larmor's formula only valid here!

We can write the Force in a magnetic field,

$$\vec{F} = \frac{q(\vec{v} \times \vec{B})}{c}. \qquad (5.1)$$

That is perpendicular to the particle velocity ($\vec{v}$) so $\vec{F} \cdot \vec{v} = 0$, $mv^2/2$ is constant, $v_\parallel$ is constant, and $|v_\perp|$ is constant.

Synchrotron Radiation (ERA Chapter 5)

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The angular velocity ω needs to be **balanced** with the the centripetal and magnetic forces:

$$m\,|\dot{v}| = m\omega^2 r = \frac{q}{c}\,|\vec{v} \times \vec{B}| = \frac{q}{c}\omega r B; \qquad (5.2)$$

Therefore, the **orbital frequency is independent of the particle speed** so long as $v \ll c$:

$$\omega_G \equiv \frac{qB}{mc}. \qquad (5.3)$$

→ in MHz (~28 Hz)

$$\left(\frac{\nu_G}{\mathrm{MHz}}\right) = 2.8\left(\frac{B}{\mathrm{gauss}}\right). \qquad (5.7)$$

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Main Takeaway:

Gyro radiation from nonrelativistic electrons are observable only in a very strong magnetic fields such that B ~ 10^{12} G !

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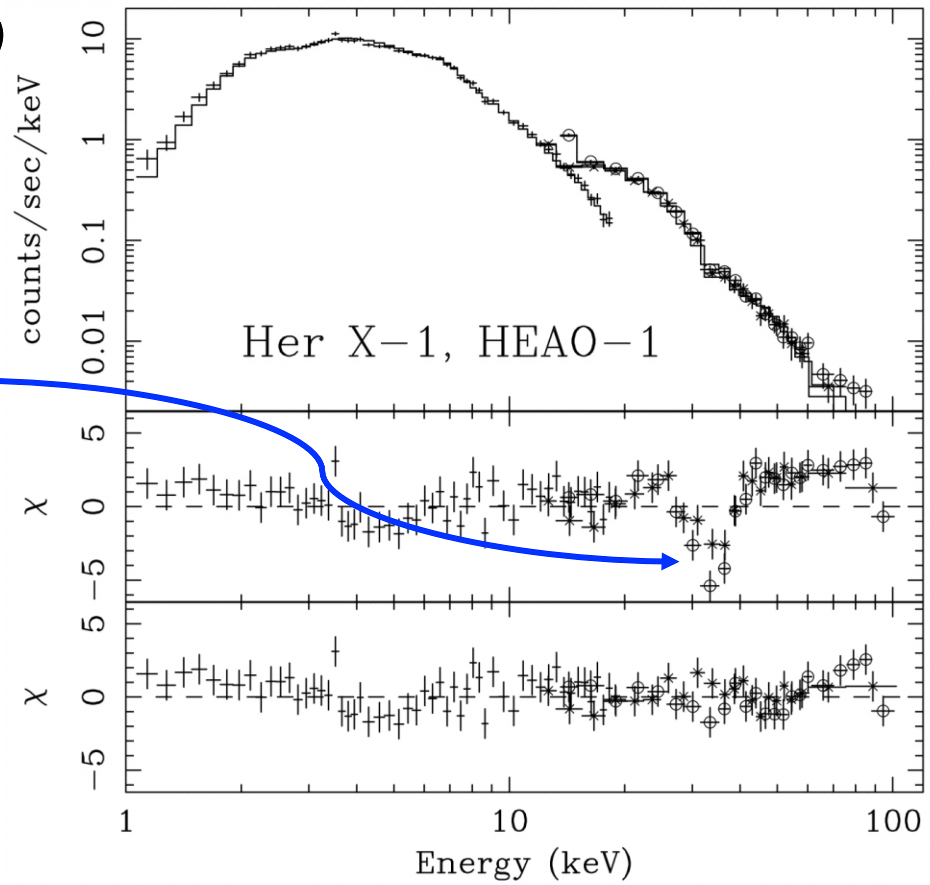
Synchrotron Radiation (ERA Chapter 5)

Magnetobremsstrahlung (5.1)

(5.1.1) **Gyro radiation**: electrons with velocities much less than the speed of light v ≪ c

Fig. 5.1 (ERA)



Gyro-resonance absorption line near 34 keV

Main Takeaway:

Gyro radiation from nonrelativistic electrons are observable only in a very strong magnetic fields such that B ~ 10^{12} G !

← The frequency of this absorption line directly measures the magnetic field strength near the Her X-1 neutron star!

$$B = \frac{2\pi\nu_G m_e c}{e}$$

$$\approx \frac{2\pi \cdot 8.2\times 10^{18}\ \text{Hz} \cdot 9.1\times 10^{-28}\ \text{g} \cdot 3\times 10^{10}\ \text{cm s}^{-1}}{4.8\times 10^{-10}\ \text{statcoul}}$$

$$\approx 2.9\times 10^{12}\ \text{gauss}.$$

Synchrotron Radiation (ERA Chapter 5)

Soo not applicable to radio frequencies !

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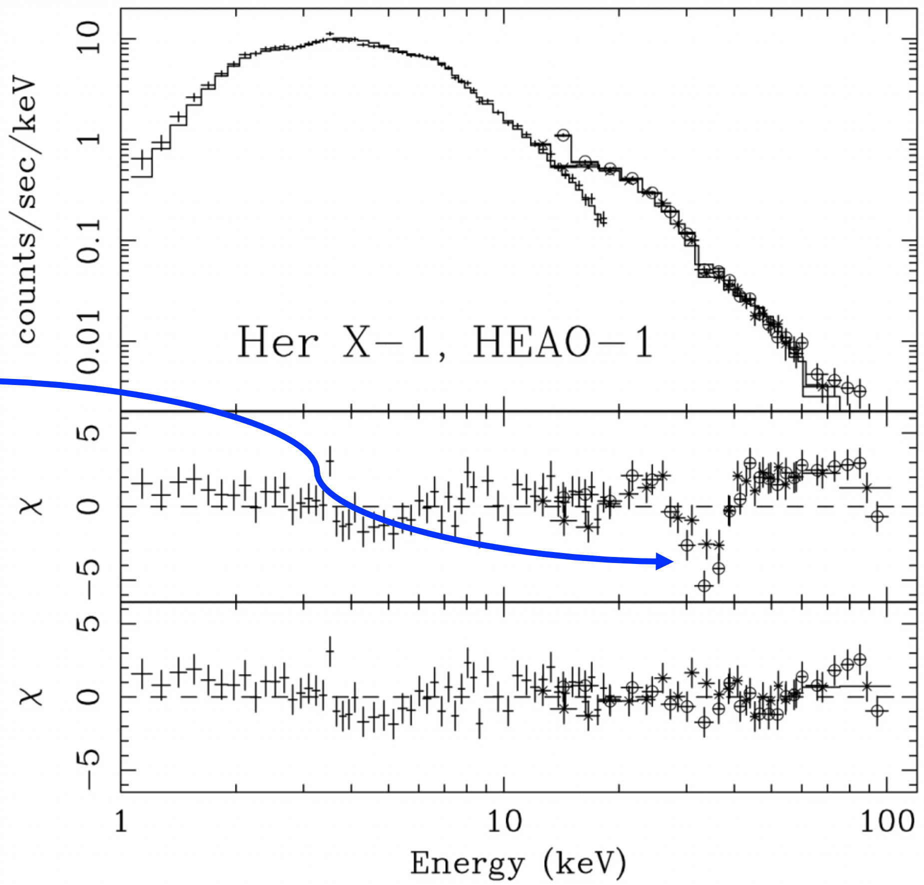

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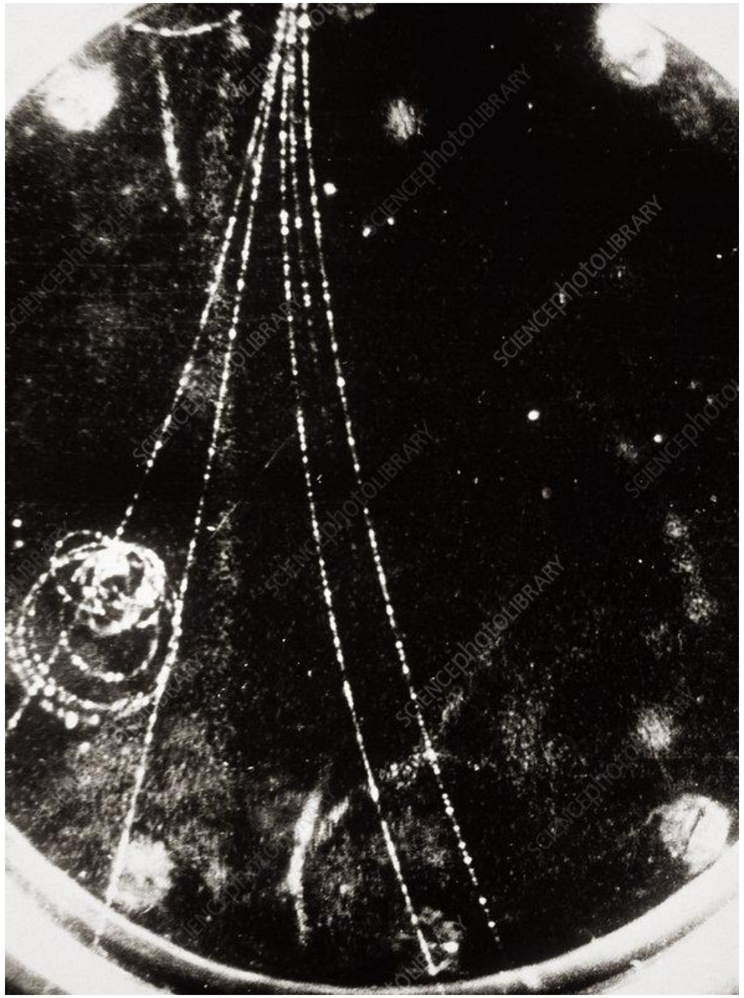
Synchrotron Radiation (ERA Chapter 5)

Synchrotron Power (5.2)

Cosmic ray electrons in the interstellar magnetic field emit **synchrotron radiation** that accounts for most of the continuum emission from our Galaxy at < 30 GHz.

Need to apply the **Lorentz transform of special relatively** to use **Larmor's formula!** *(see Appendix C)*



Electrons and positrons produced by a cosmic ray which has interacted in the wall of the cloud chamber

Synchrotron Radiation (ERA Chapter 5)

Synchrotron Power (5.2)

Lorentz transforms (Appendix C for derivation) relates the coordinates (x, y, z, t) in the unprimed inertial frame and the coordinate (x', y', x', t') in the primed frame moving with velocity v in the x-direction.
They are:

$$x = \gamma(x' + vt'), \quad y = y', \quad z = z', \quad t = \gamma(t' + \beta x'/c), \tag{5.12}$$

$$x' = \gamma(x - vt), \quad y' = y, \quad z' = z, \quad t' = \gamma(t - \beta x/c), \tag{5.13}$$

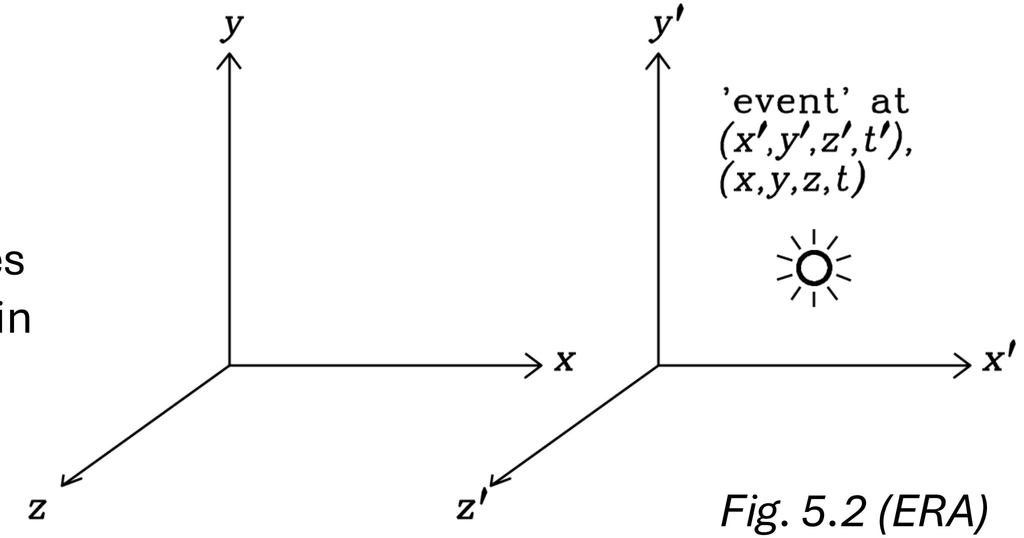


Fig. 5.2 (ERA)

Where,

$$\beta \equiv v/c \qquad \gamma \equiv (1-\beta^2)^{-1/2} \qquad \text{(5.14 \& 5.15)}$$

And γ is the **Lorentz factor!** Note the differential form is:

$$\Delta x = \gamma(\Delta x' + v\Delta t'), \quad \Delta y = \Delta y', \quad \Delta z = \Delta z', \quad \Delta t = \gamma(\Delta t' + \beta \Delta x'/c), \tag{5.16}$$

$$\Delta x' = \gamma(\Delta x - v\Delta t), \quad \Delta y' = \Delta y, \quad \Delta z' = \Delta z, \quad \Delta t' = \gamma(\Delta t - \beta \Delta x/c). \tag{5.17}$$

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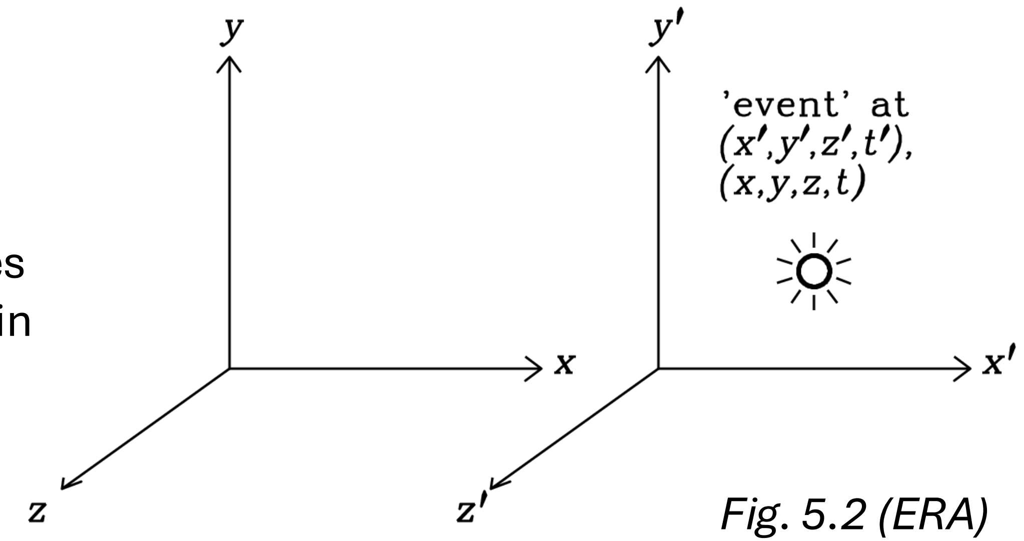

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Where,

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This factor will be important to introduce into power spectrum, observed frequencies, emission coefficient, etc.,!

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Synchrotron Radiation (ERA Chapter 5)

Synchrotron Power (5.2)

(5.2.2) Relativistic Masses

Cosmic-ray electrons have E >> $E_0 = m_e c^2$ >> 0.51 MeV and are ultrarelativistic:

$$E_0 = m_e c^2 = 9.1 \times 10^{-28} \text{ g} \cdot (3 \times 10^{10} \text{ cm s}^{-1})^2 = 8.2 \times 10^{-7} \text{ erg} \qquad (5.19)$$

$$= \frac{8.2 \times 10^{-7} \text{ erg}}{1.60 \times 10^{-12} \text{ erg (eV)}^{-1}} = 5.1 \times 10^5 \text{ eV} = 0.51 \text{ MeV}. \qquad (5.20)$$

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Remember: orbital frequency from Gyro radiation,

$$\boxed{\omega_G \equiv \frac{qB}{mc}.} \quad (5.3) \quad \rightarrow \text{becomes} \quad \omega_B = \frac{eB}{(\gamma m_e)\, c} = \frac{\omega_G}{\gamma}. \quad (5.21)$$

Which is ***not promising for the production of observable synchrotron radiation***: the high observed masses $m = \gamma m_e$ of relativistic electrons reduce their orbital frequencies and accelerations to extremely low values!

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We will see two effects that explain the strong observable synchrotron emission at radio frequencies:

1) Total radiated power in the observer's frame is proportional to γ^2 (a large number)

2) Relativistic beaming turns the low frequency sinusoidal radiation in electron frame into series of extremely sharp pulses containing power at much higher frequency in the observer's frame