Emission Mechanisms

Spectral Lines (ERA Chap. 7)

Free-Free (ERA Chap. 4)

Synchrotron (ERA Chap. 5)

Pulsars (ERA Chap. 6)

Contact: sscibell@nrao.edu

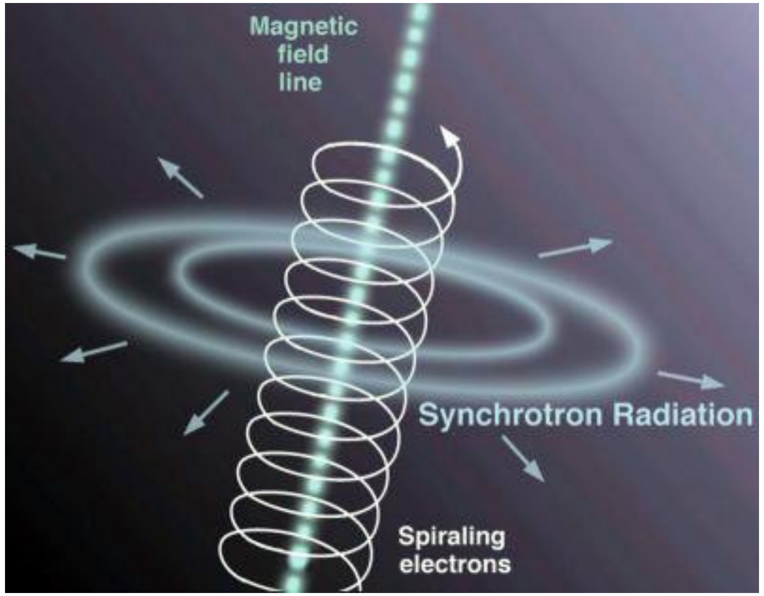
Synchrotron Radiation (ERA Chapter 5)

Magnetobremsstrahlung (5.1)

Remember: the lightest particles are accelerated more so **electrons account for virtually all of the radiation observed**

Types:

- **Gyro radiation**: electrons with velocities much less than the speed of light $v \ll c$
- **Cyclotron radiation:** mildly relativistic electrons w/ kinetic energy comparable to the rest mass $m_e c^2$
- **Synchrotron radiation:** ultra relativistic electrons with kinetic energy $\gg$ rest mass $m_e c^2$



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- Accounts for **most of the radio emission from** 'Active Galactic Nuclei' or **AGNs** that are thought to be powered by supermassive black holes in galaxies and quasars
- **Dominates radio continuum emission of star forming galaxies at ν < 30 GHz**
- Magnetosphere of Jupiter is also a synchrotron source
- Synchrotron radiation also presents as...
  - optical emission from Crab Nebula
  - optical jet of M87
  - optical → X-ray emission of quasars

Synchrotron Radiation (ERA Chapter 5)

Magnetobremsstrahlung (5.1)

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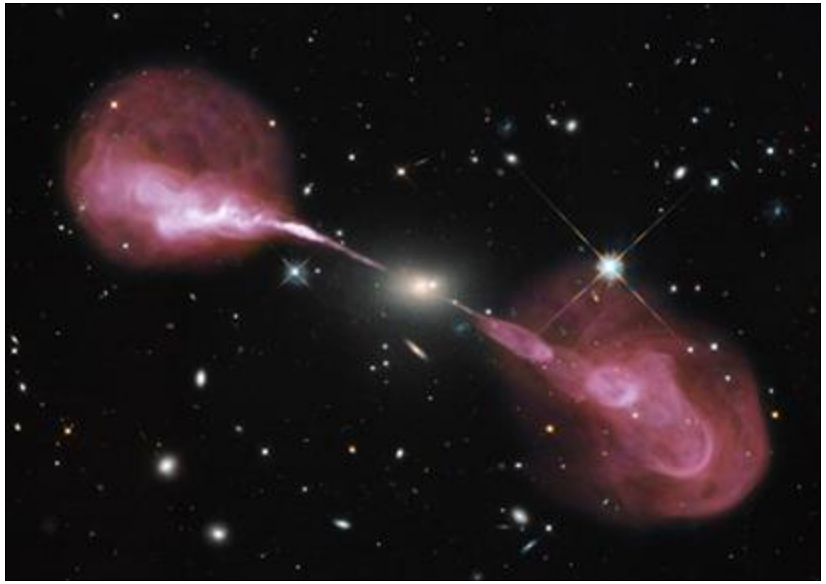


Synchrotron Radiation (ERA Chapter 5)

Magnetobremsstrahlung (5.1)

(5.1.1) **Gyro radiation**: electrons with velocities much less than the speed of light $v \ll c$

→ **Larmor's formula** only valid here!

We can write the Force in a magnetic field,

$$\vec{F} = \frac{q(\vec{v} \times \vec{B})}{c}. \quad (5.1)$$

Main Takeaway:

Gyro radiation from nonrelativistic electrons are observable only in a very strong magnetic fields such that $B \sim 10^{12}$ G !

That is perpendicular to the particle velocity ($\vec{v}$) so $\vec{F} \cdot \vec{v} = 0$, $mv^2/2$ is constant, $v_{\parallel}$ is constant, and $|v_{\perp}|$ is constant.
The angular velocity ω needs to be **balanced** with the the centripetal and magnetic forces:

$$m|\dot{v}| = m\omega^2 r = \frac{q}{c}|\vec{v} \times \vec{B}| = \frac{q}{c}\omega r B; \quad (5.2)$$

Therefore, the **orbital frequency is independent of the particle speed** so long as $v \ll c$:

$$\omega_G \equiv \frac{qB}{mc}. \quad (5.3)$$

→ in MHz (~28 Hz)

$$\left(\frac{\nu_G}{\text{MHz}}\right) = 2.8\left(\frac{B}{\text{gauss}}\right). \quad (5.7)$$

Synchrotron Radiation (ERA Chapter 5)

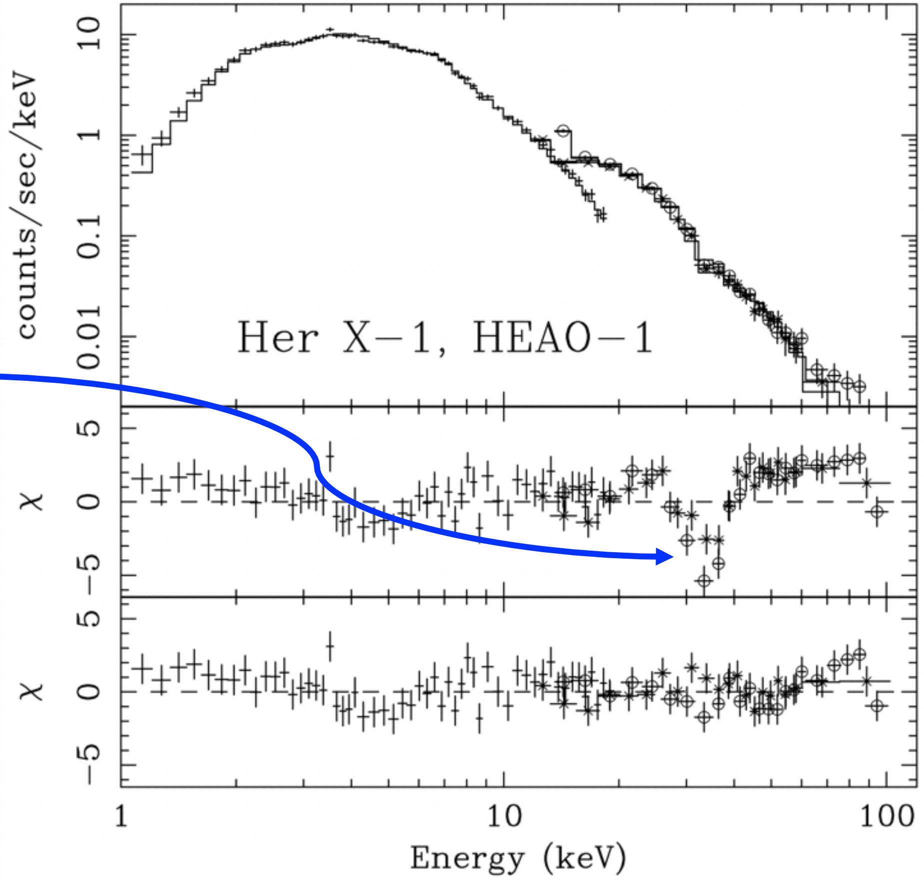
Soo not applicable to radio frequencies !

Magnetobremsstrahlung (5.1)

(5.1.1) **Gyro radiation**: electrons with velocities much less than the speed of light $v \ll c$

Fig. 5.1 (ERA)

Gyro-resonance absorption line near 34 keV



Main Takeaway:

Gyro radiation from nonrelativistic electrons are observable only in a very strong magnetic fields such that B ~ 10^{12} G !

← The frequency of this absorption line directly measures the magnetic field strength near the Her X-1 neutron star!

$$B = \frac{2\pi\nu_G m_e c}{e}$$

$$\approx \frac{2\pi \cdot 8.2\times10^{18}\ \text{Hz} \cdot 9.1\times10^{-28}\ \text{g} \cdot 3\times10^{10}\ \text{cm s}^{-1}}{4.8\times10^{-10}\ \text{statcoul}}$$

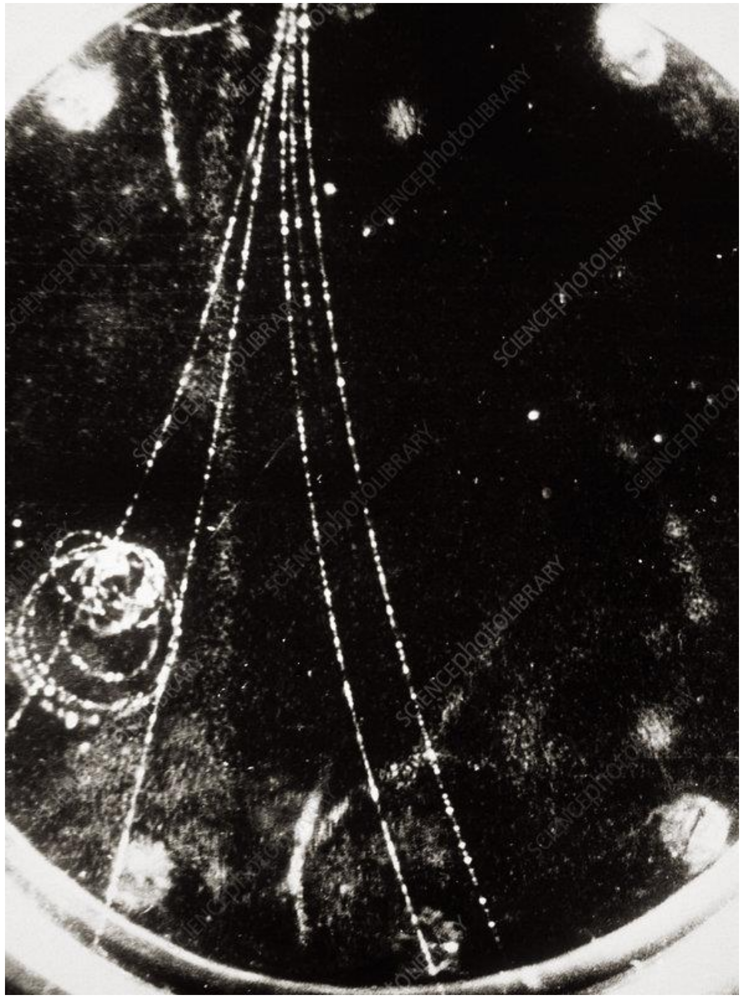
$$\approx 2.9\times10^{12}\ \text{gauss.}$$

Synchrotron Radiation (ERA Chapter 5)

Synchrotron Power (5.2)

Cosmic ray electrons in the interstellar magnetic field emit **synchrotron radiation** that accounts for most of the continuum emission from our Galaxy at < 30 GHz.

Need to apply the **Lorentz transform of special relatively** to use **Larmor's formula!** *(see Appendix C)*



Electrons and positrons produced by a cosmic ray which has interacted in the wall of the cloud chamber

Synchrotron Radiation (ERA Chapter 5)

Synchrotron Power (5.2)

Lorentz transforms (Appendix C for derivation) relates the coordinates (x, y, z, t) in the unprimed inertial frame and the coordinate (x', y', x', t') in the primed frame moving with velocity v in the x-direction.
They are:

$$x = \gamma(x' + vt'), \quad y = y', \quad z = z', \quad t = \gamma(t' + \beta x'/c), \tag{5.12}$$

$$x' = \gamma(x - vt), \quad y' = y, \quad z' = z, \quad t' = \gamma(t - \beta x/c), \tag{5.13}$$

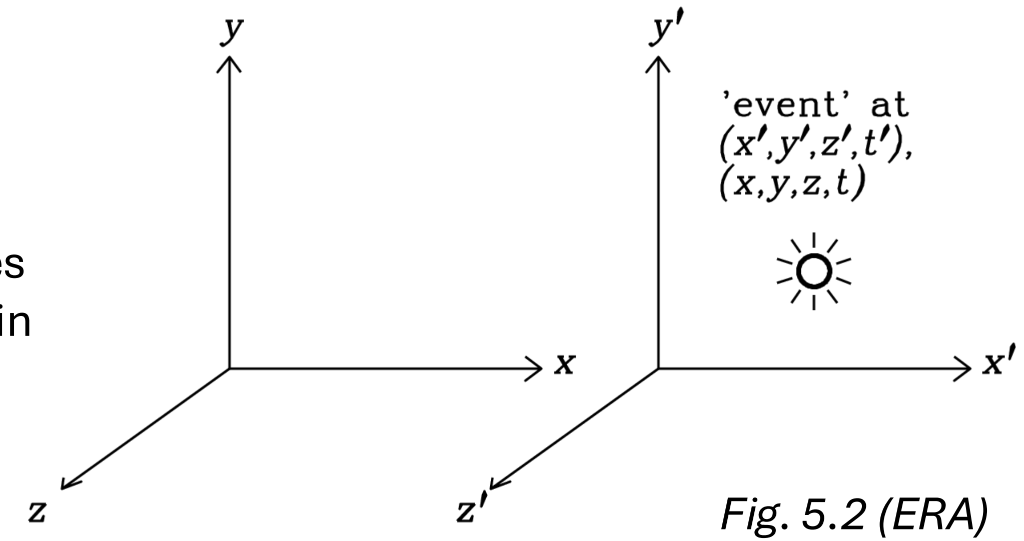


Fig. 5.2 (ERA)

Where,

$$\beta \equiv v/c \qquad \gamma \equiv (1-\beta^2)^{-1/2} \tag{5.14 \& 5.15}$$

This factor will be important to introduce into power spectrum, observed frequencies, emission coefficient, etc.,!

And γ is the **Lorentz factor!** Note the differential form is:

$$\Delta x = \gamma(\Delta x' + v\Delta t'), \quad \Delta y = \Delta y', \quad \Delta z = \Delta z', \quad \Delta t = \gamma(\Delta t' + \beta \Delta x'/c), \tag{5.16}$$

$$\Delta x' = \gamma(\Delta x - v\Delta t), \quad \Delta y' = \Delta y, \quad \Delta z' = \Delta z, \quad \Delta t' = \gamma(\Delta t - \beta \Delta x/c). \tag{5.17}$$

Synchrotron Radiation (ERA Chapter 5)

Synchrotron Power (5.2)

(5.2.2) Relativistic Masses

Cosmic-ray electrons have E >> $E_0 = m_e c^2$ >> 0.51 MeV and are ultrarelativistic:

$$E_0 = m_e c^2 = 9.1 \times 10^{-28} \text{ g} \cdot (3 \times 10^{10} \text{ cm s}^{-1})^2 = 8.2 \times 10^{-7} \text{ erg} \quad (5.19)$$

$$= \frac{8.2 \times 10^{-7} \text{ erg}}{1.60 \times 10^{-12} \text{ erg (eV)}^{-1}} = 5.1 \times 10^5 \text{ eV} = 0.51 \text{ MeV}. \quad (5.20)$$

Remember: orbital frequency from Gyro radiation,

$$\boxed{\omega_G \equiv \frac{qB}{mc}.} \quad (5.3) \quad \rightarrow \text{becomes} \quad \omega_B = \frac{eB}{(\gamma m_e) c} = \frac{\omega_G}{\gamma}. \quad (5.21)$$

Which is ***not promising for the production of observable synchrotron radiation***: the high observed masses $m = \gamma m_e$ of relativistic electrons reduce their orbital frequencies and accelerations to extremely low values!

The orbital frequency and orbital size is very large … electrons go around slow and in 'loopy' orbits

$$\nu_B \equiv \frac{\omega_B}{2\pi} \quad (5.22)$$

$$\approx 28 \times 10^{-5} \text{ Hz}$$

$$\approx 1 \text{ cycle per hour.}$$

$$r \approx \frac{c}{\omega_B} \quad (5.23)$$

$$\approx \frac{3 \times 10^{10} \text{ cm s}^{-1}}{2\pi \cdot 28 \times 10^{-5} \text{ Hz}}$$

$$\approx 1.7 \times 10^{13} \text{ cm} \approx 1 \text{ AU.}$$

Synchrotron Radiation (ERA Chapter 5)

Synchrotron Power (5.2)

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We will see two effects that explain the strong observable synchrotron emission at radio frequencies:

1) Total radiated power in the observer's frame is proportional to γ^2 (a large number)

2) Relativistic beaming turns the low frequency sinusoidal radiation in electron frame into series of extremely sharp pulses containing power at much higher frequency in the observer's frame

Synchrotron Radiation (ERA Chapter 5)

Synchrotron Power (5.2)

(5.2.3) ***Synchrotron Power Radiated by a single electron***

Larmor's Formula: $P = \frac{2q^2\dot{v}^2}{3c^3}$ (4.1)

See text for derivation with Larmor's formula using Lorentz transforms to get to the power radiated by a single electron moving with constant **pitch angle α** between the electron velocity, $\vec{v}$, and the magnetic field, $\vec{B}$:

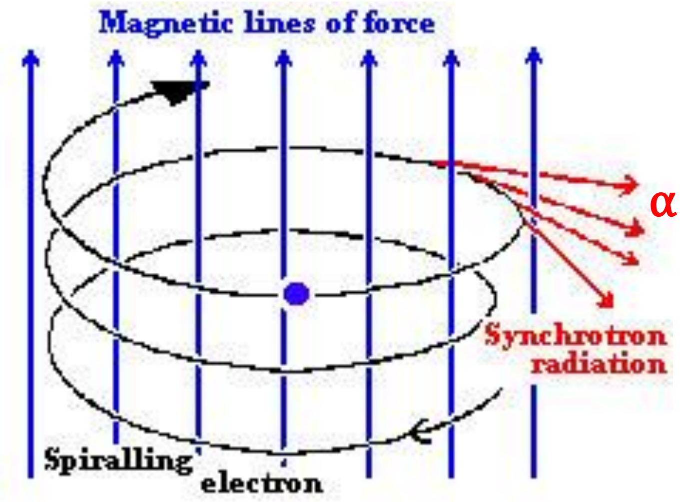
$$P = \frac{2e^2}{3c^3}\gamma^2\frac{e^2B^2}{m_e^2c^2}v^2\sin^2\alpha. \qquad (5.32)$$

Where the radiated power *P'* was transformed to the electron frame, *P*

$$P = P' = \frac{2e^2a_\perp^2\gamma^4}{3c^3} \qquad (a_\parallel = 0). \qquad (5.29)$$

And the magnetic acceleration is balanced in a circular orbit,

$$a_\perp \equiv \frac{dv_\perp}{dt} = \omega_B v_\perp = \frac{eBv_\perp}{\gamma m_e c} = \frac{eBv\sin\alpha}{\gamma m_e c}, \qquad (5.30 \ \& \ 5.31)$$



Synchrotron Radiation (ERA Chapter 5)

Synchrotron Power (5.2)

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$$P = \frac{2e^2}{3c^3}\gamma^2 \frac{e^2 B^2}{m_e^2 c^2} v^2 \sin^2 \alpha. \qquad (5.32)$$

Often written in terms of the **Thomson cross section** of an electron:

$$\sigma_T \equiv \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 \approx 6.65 \times 10^{-25}\ \mathrm{cm}^2 \qquad (5.33\ \&\ 5.34)$$

Synchrotron Radiation (ERA Chapter 5)

Synchrotron Power (5.2)

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And the **magnetic energy density:**

$$U_B = \frac{B^2}{8\pi}. \qquad (5.35)$$

Synchrotron Radiation (ERA Chapter 5)

Synchrotron Power (5.2)

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$$\rightarrow\ P = 2\sigma_T \beta^2 \gamma^2 c\, U_B \sin^2 \alpha. \qquad (5.37)$$

Synchrotron power radiated by a **single electron** depends only on physical constants, **the square of the electron kinetic energy (via γ)**, the magnetic energy density, U_B, and pitch angle, α

Synchrotron Radiation (ERA Chapter 5)

Synchrotron Power (5.2)

(5.2.3) ***Synchrotron Power Radiated by a single electron***

The **average synchrotron power <P>** per electron in an ensemble of electrons having the same Lorentz factor and isotopically distributed **pitch angles α**

$$\langle P \rangle = \frac{4}{3}\sigma_T \beta^2 \gamma^2 c U_B. \quad (5.42)$$

Relativistic electrons in radio sources can have lifetimes of thousands of millions of years before losing their ultrarelativistic energies via synchrotron radiation or other processes -- the distribution of their pitch angles gradually becomes random and isotropic -- **$\sin^2 \alpha$ becomes factor of 2/3**

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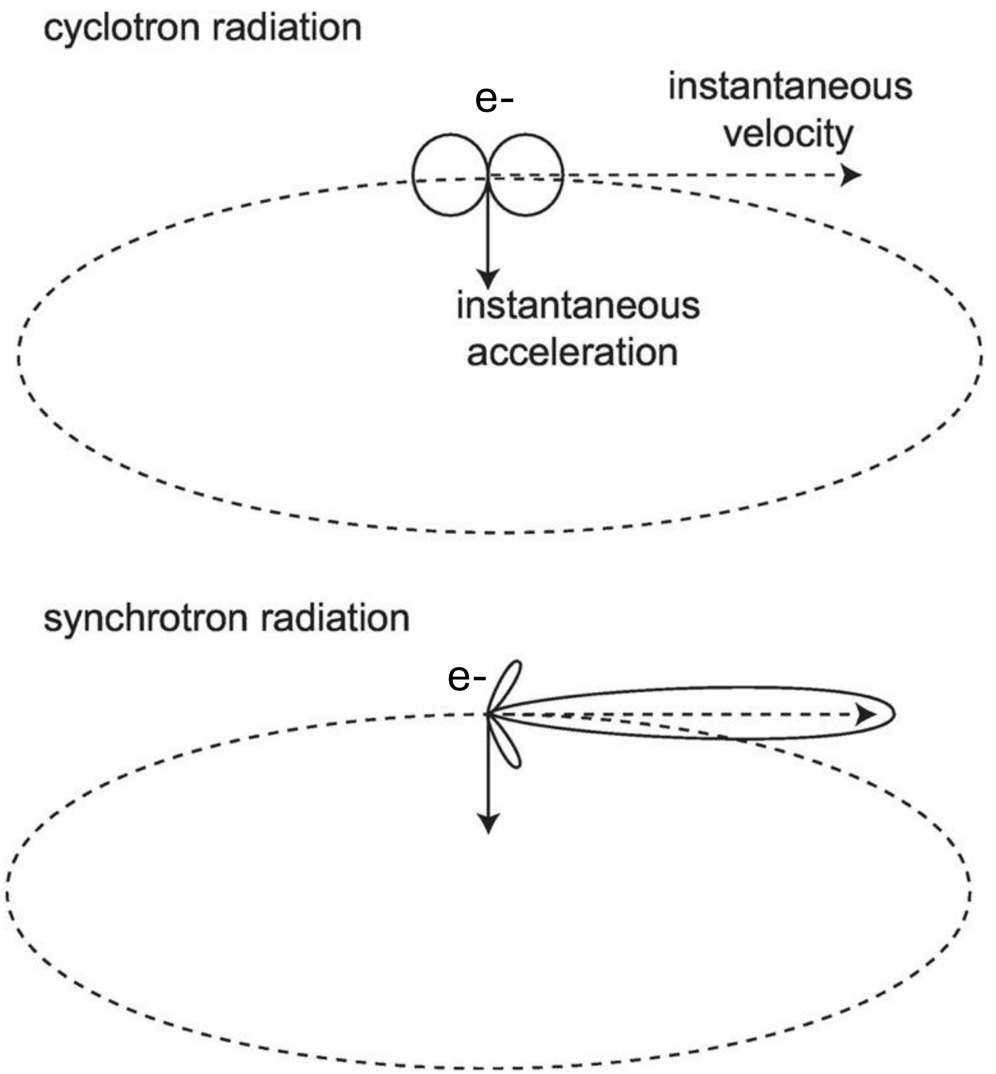
Synchrotron Radiation (ERA Chapter 5)

Synchrotron Spectra (5.3)

(5.3.1) Synchrotron Spectrum of a Single Electron



As the electron goes around its orbit, we see the following effect →

"Flash" from electron going around its orbit is going almost as fast as the speed of light so we get two main effects:

1) Pulse is amplified
2) Pulse is shortened

Synchrotron Radiation (ERA Chapter 5)

Synchrotron Spectra (5.3)

*(5.3.1) **Synchrotron Spectrum of a Single Electron***

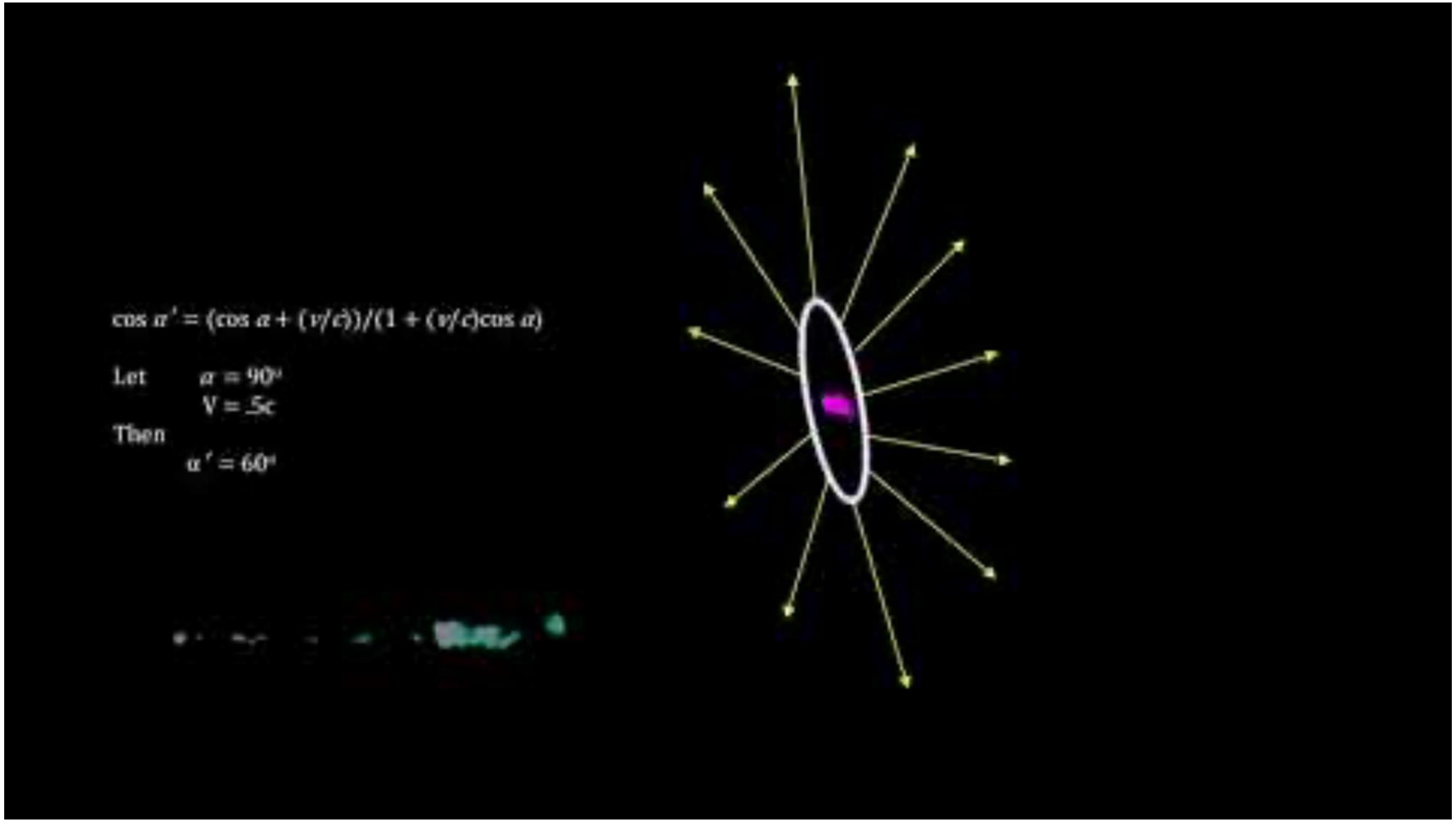
Here we talk about **relativistic aberration/beaming** to show that synchrotron radiation appears at frequencies much higher than $\omega_B = \omega_G/\gamma$

Fig. 5.3 (ERA) — Relativistic aberration transforms the dipole power pattern of Larmor radiation in the electron rest frame (dotted curve) into a narrow searchlight beam in the observer's frame. The solid curve is the transformed pattern for $\gamma = 5$. The observed angle between the nulls of the forward beam falls to $\Delta\theta = 2\arcsin(1/\gamma) \approx 2/\gamma$ in the limit $\gamma \gg 1$.

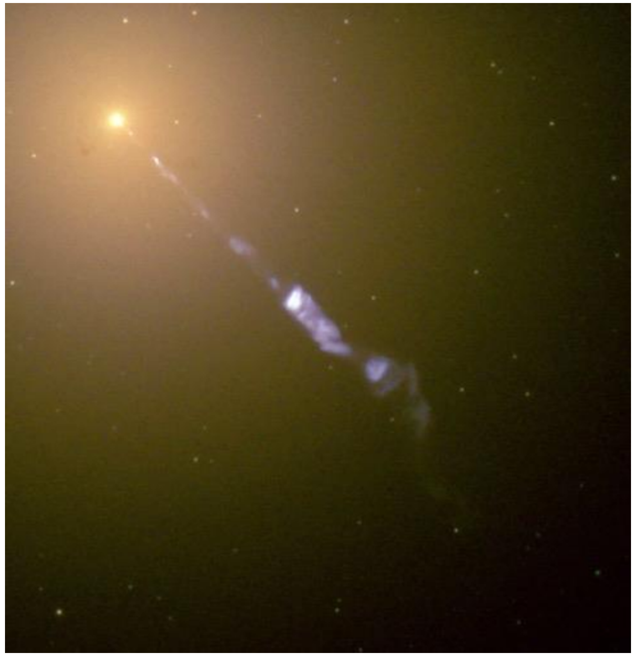
Photon beaming follows directly from relativistic velocity addition equations (see text, won't go over here!)

e.g., 10 GeV electron where γ ~ 2 x 10^4 so $2/\gamma$ ~ 10^{-4} rad or beaming angle of ~ 20'' !

Relativistic Beaming

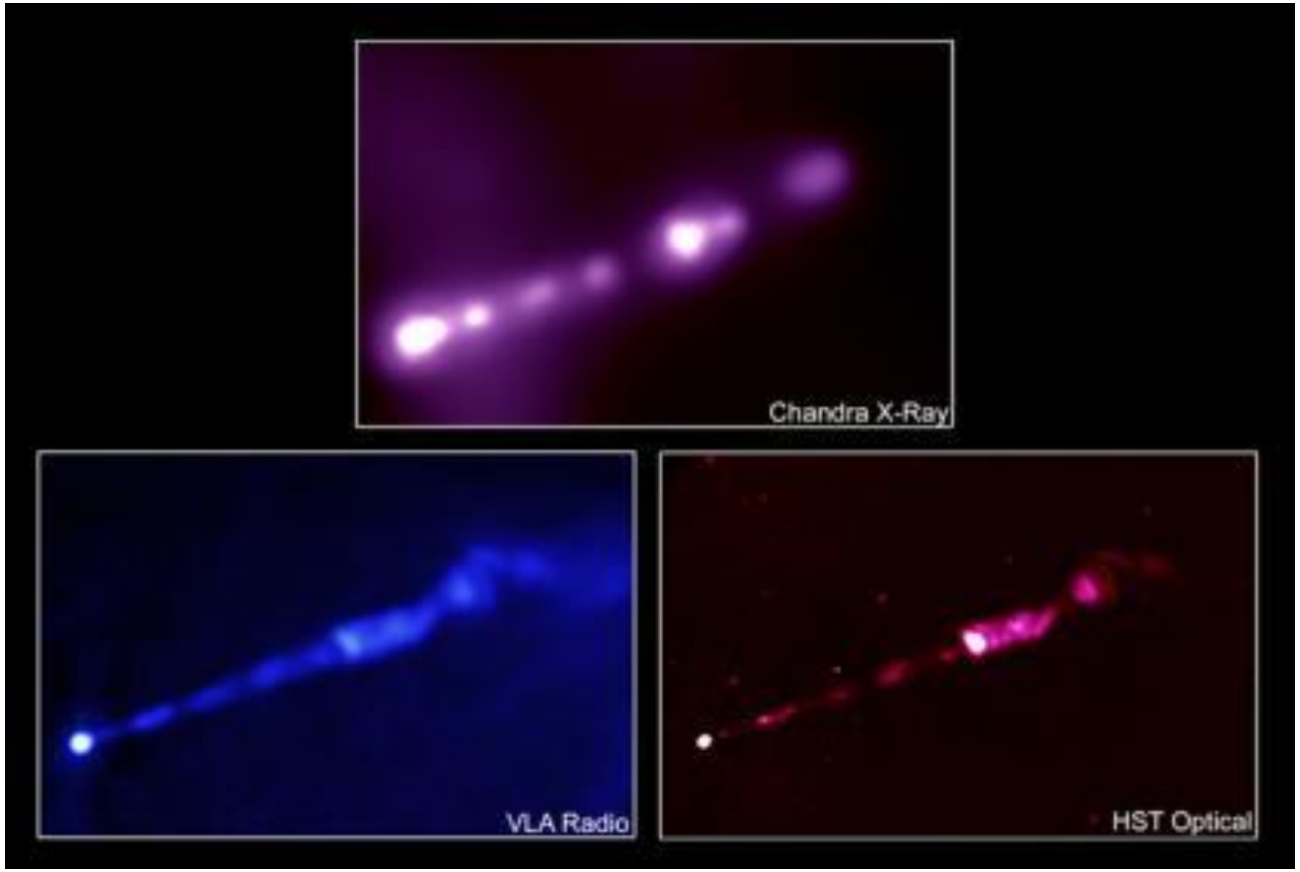


David Butler

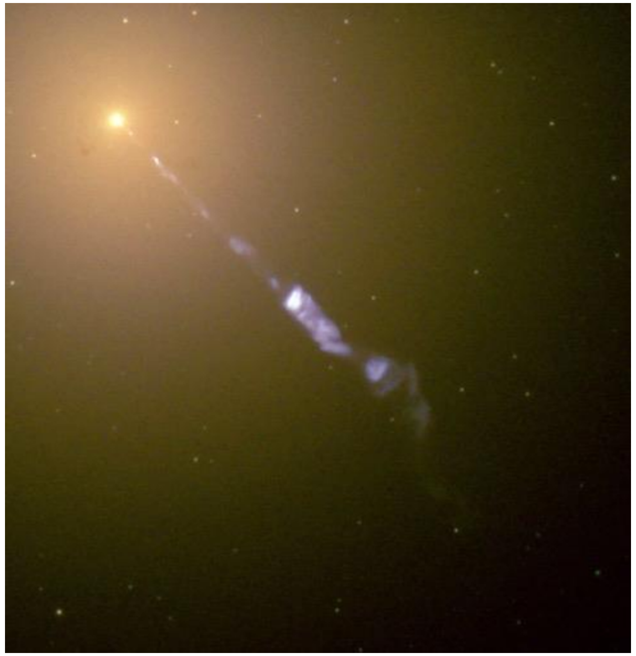


Example: M87 Jet
Credit: NASA and the
Hubble Heritage Team (STScI/AURA)

Relativistic Beaming: M87 Jet



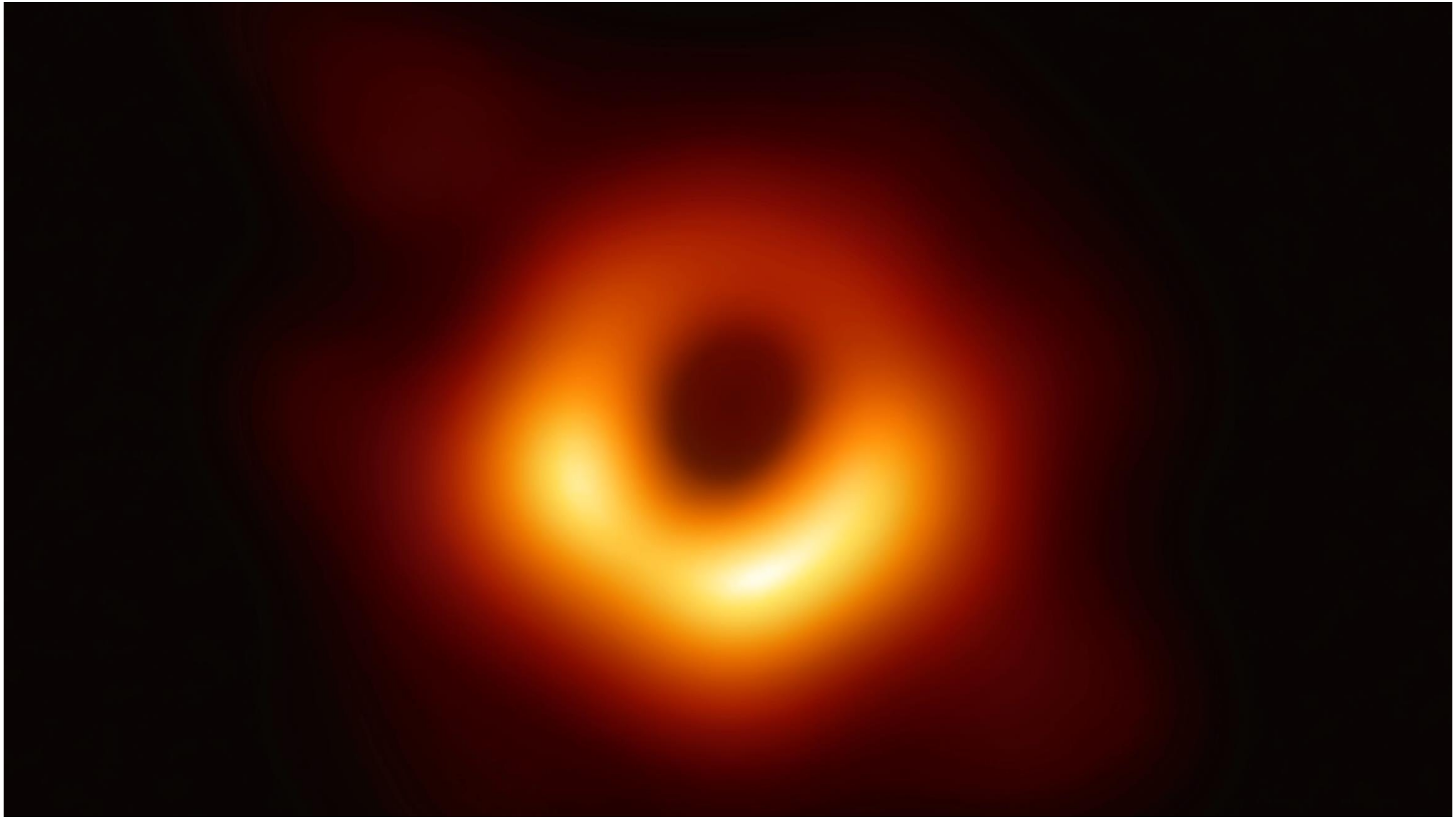
Credit: X-ray: NASA/CXC/MIT/H.Marshall et al. Radio: F. Zhou, F.Owen (NRAO), J.Biretta (STScI) Optical: NASA/STScI/UMBC/E.Perlman et al.



Example: M87 Jet
Credit: NASA and the
Hubble Heritage Team (STScI/AURA)

M87 Jets compared to EHT image

For scale !→



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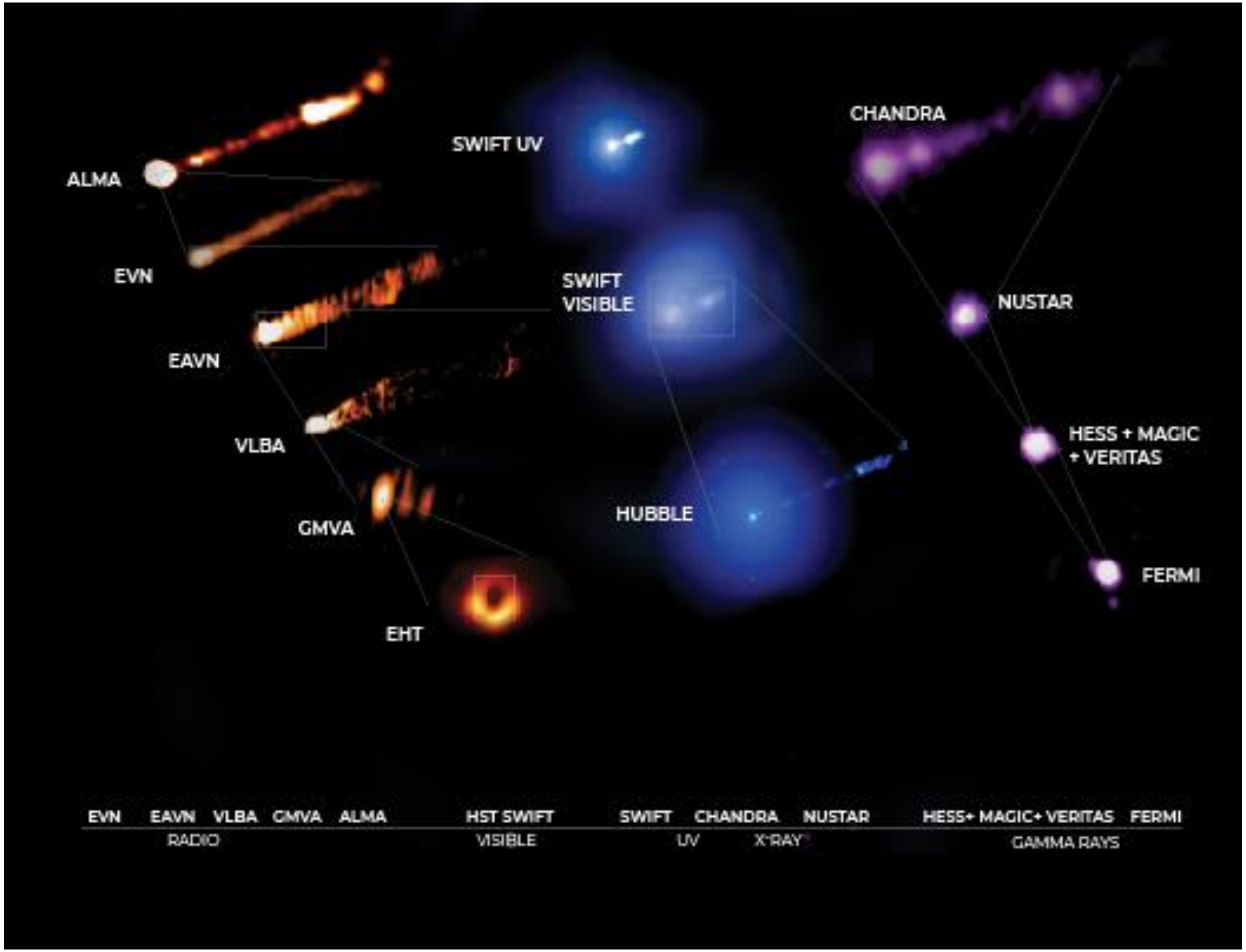

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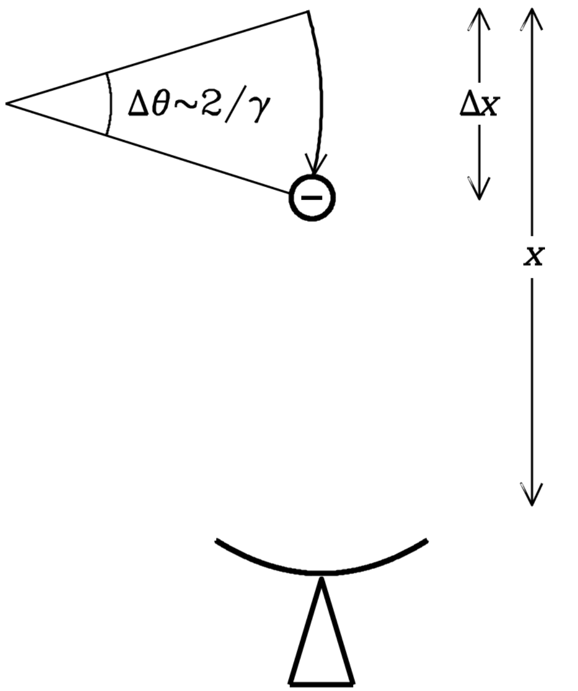
Synchrotron Radiation (ERA Chapter 5)

Synchrotron Spectra (5.3)

(5.3.1) ***Synchrotron Spectrum of a Single Electron***

Fig. 5.4 (ERA)



Duration of a pulse:

$$\Delta t_{\rm p} = t(\text{end of observed pulse}) - t(\text{start of observed pulse}) \quad (5.54)$$

$$= \frac{\Delta x}{v} + \frac{(x - \Delta x)}{c} - \frac{x}{c}. \quad (5.55)$$

the observed pulse duration:

$$\Delta t_{\rm p} = \frac{\Delta x}{v} - \frac{\Delta x}{c} = \frac{\Delta x}{v}\left(1 - \frac{v}{c}\right) \ll \frac{\Delta x}{v} = \Delta t \quad (5.56)$$

In the limit v → c:

$$\left(1 - \frac{v}{c}\right) = \left(1 - \frac{v}{c}\right)\frac{1 + v/c}{1 + v/c} = \frac{1 - v^2/c^2}{1 + v/c} \approx \frac{\gamma^{-2}}{2} = \frac{1}{2\gamma^2} \quad (5.57)$$

The **observed pulse duration is shortened by a factor (1−v/c).**
Plugging into 5.56:

$$\Delta t_{\rm p} = \frac{\Delta t}{2\gamma^2} = \frac{\Delta x}{v}\frac{1}{2\gamma^2} = \frac{\Delta\theta}{\omega_B}\frac{1}{2\gamma^2}. \quad (5.58)$$

The **observed pulse duration Δt_p is much less than the time Δt the electron needs to move a distance** Δx because in the observer's frame the electron nearly keeps up with its own radiation!

Synchrotron Radiation (ERA Chapter 5)

Synchrotron Spectra (5.3)

(5.3.1) ***Synchrotron Spectrum of a Single Electron***

In terms of angular frequency and pitch angle α,

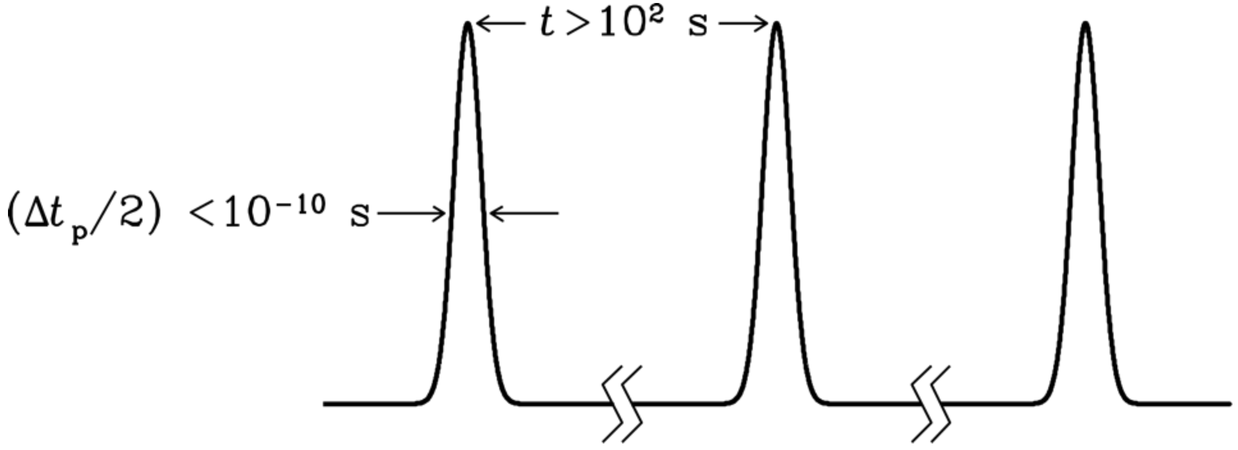
$$\Delta t_p = \frac{1}{\gamma^2 \omega_G \sin\alpha}, \qquad (5.60)$$

The power received as a function of time is very **spiky**, which means that the FT of this is nearly a continuous series of spikes in the frequency domain

Adjacent spikes are separated in frequency by only:

$$\Delta\nu = \frac{\nu_G}{\gamma} < 10^{-3} \text{ Hz.} \qquad (5.64)$$

Fig. 5.5 (ERA)



e.g., If $\gamma \approx 10^4$ and B ~ 10μG, the half width of each pulse is $\Delta t_p/2 < 10^{-10}$ s and the spacing between pulses is $\gamma/\nu_G > 10^2$ s

Synchrotron Radiation (ERA Chapter 5)

Synchrotron Spectra (5.3)

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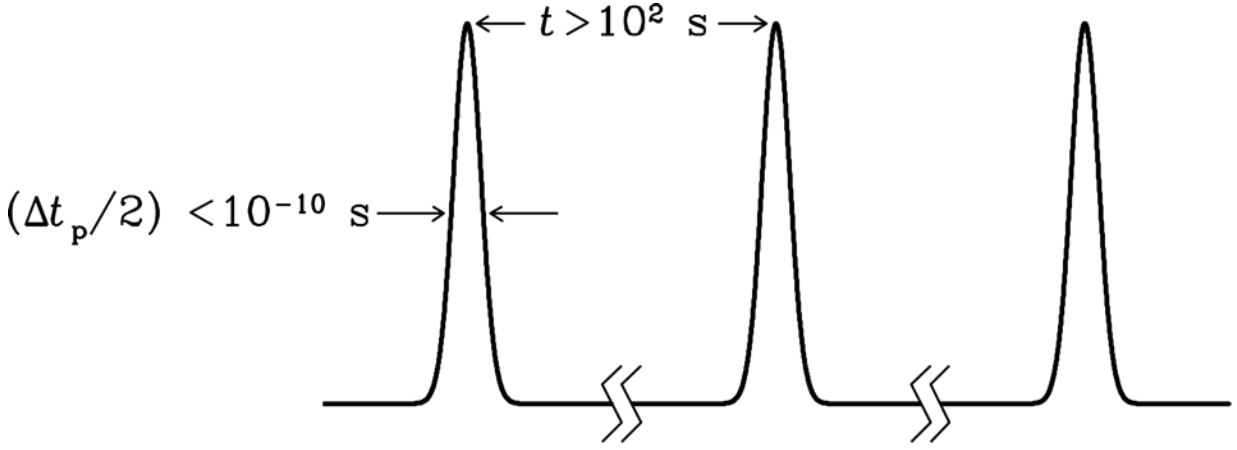


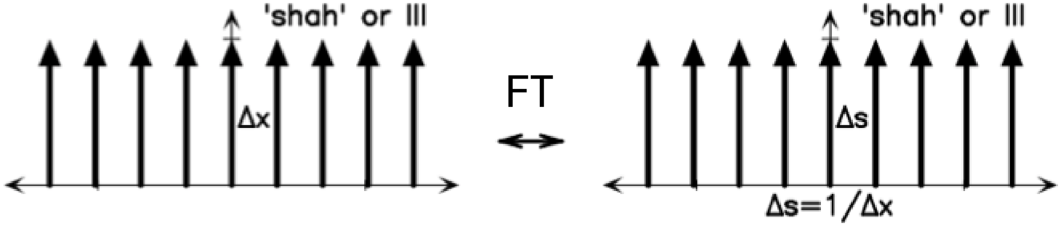
e.g., If $\gamma \approx 10^4$ and B ~ 10μG, the half width of each pulse is $\Delta t_p/2 < 10^{-10}$ s and the spacing between pulses is $\gamma/\nu_G > 10^2$ s

Remember what the Fourier transform of a time series of pulses is?

Synchrotron Radiation (ERA Chapter 5)

Synchrotron Spectra (5.3)

(5.3.1) Synchrotron Spectrum of a Single Electron



From Fig. A.1 (ERA)

The pulse train is the convolution of the individual pulse profile with the **shah function**

The Fourier transforms:

$$\text{III}\left(\frac{t\nu_G}{\gamma}\right) \qquad (5.62) \qquad \text{III}\left(\frac{\nu\gamma}{\nu_G}\right), \qquad (5.63)$$

Although this is not formally a continuous spectrum, the **frequency shifts caused by even tiny fluctuations** in electron energy, magnetic field strength, or pitch angle **cause frequency shifts much larger than Δv**, so, the spectrum of synchrotron radiation is **effectively continuous**.

Synchrotron Radiation (ERA Chapter 5)

Synchrotron Spectra (5.3)

*(5.3.1) **Synchrotron Spectrum of a Single Electron***

So, what does the spectrum actually look like?

The synchrotron spectrum of a single electron is fairly flat at low frequencies and tapers off at frequencies above,

$$\nu_{\max} \approx \frac{1}{2\Delta t_p} \approx \pi\gamma^2\nu_G \sin\alpha \propto \gamma^2 B_\perp. \qquad (5.65)$$

The continuous distribution looks more like a power-law that should be plotted in log space →

And the **formal solution to the power spectrum** is,

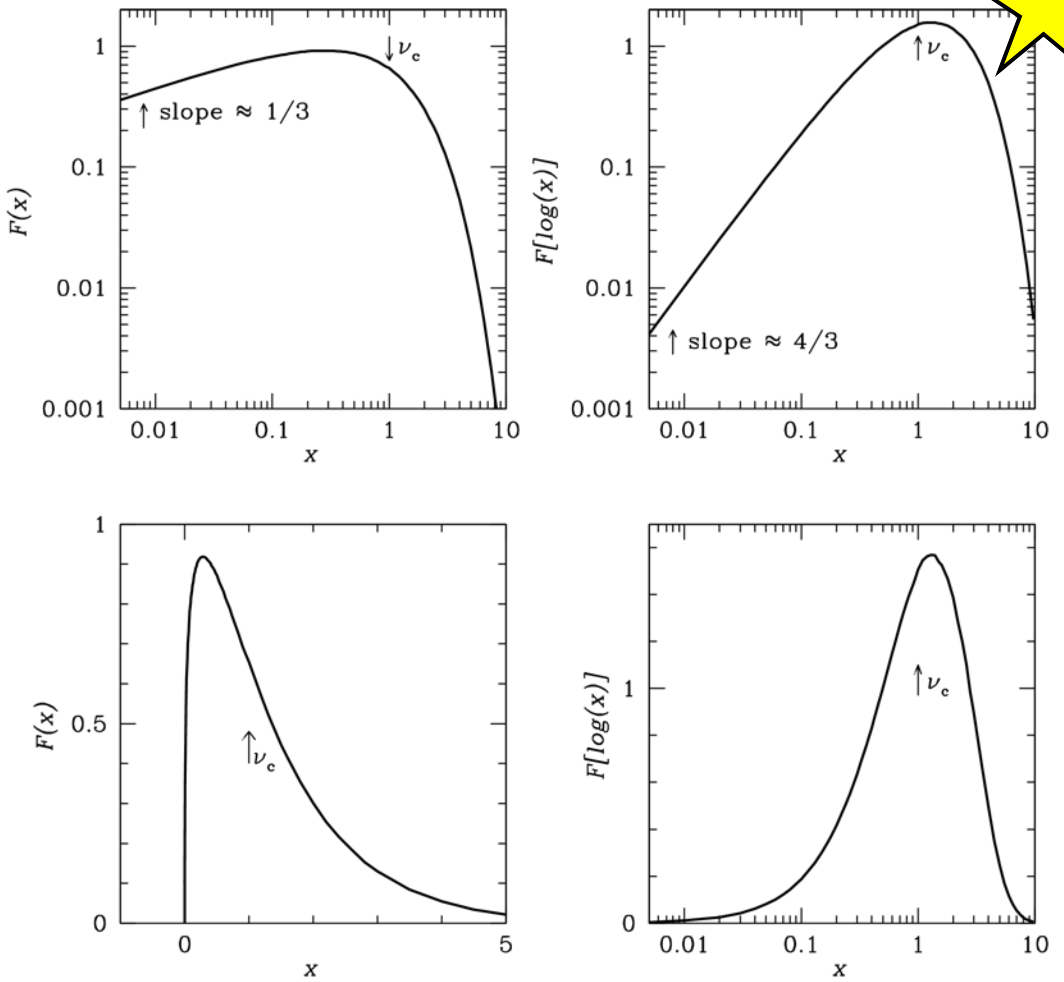
$$P(\nu) = \frac{\sqrt{3}e^3 B \sin\alpha}{m_e c^2}\left(\frac{\nu}{\nu_c}\right)\int_{\nu/\nu_c}^{\infty} K_{5/3}(\eta)\, d\eta, \qquad (5.66)$$

Where,

$$\nu_c = \frac{3}{2}\gamma^2\nu_G \sin\alpha. \qquad (5.67)$$

Fig. 5.6 (ERA)

Four different ways to plot the synchrotron spectrum of a single electron:



Synchrotron Radiation (ERA Chapter 5)

Synchrotron Spectra (5.3)

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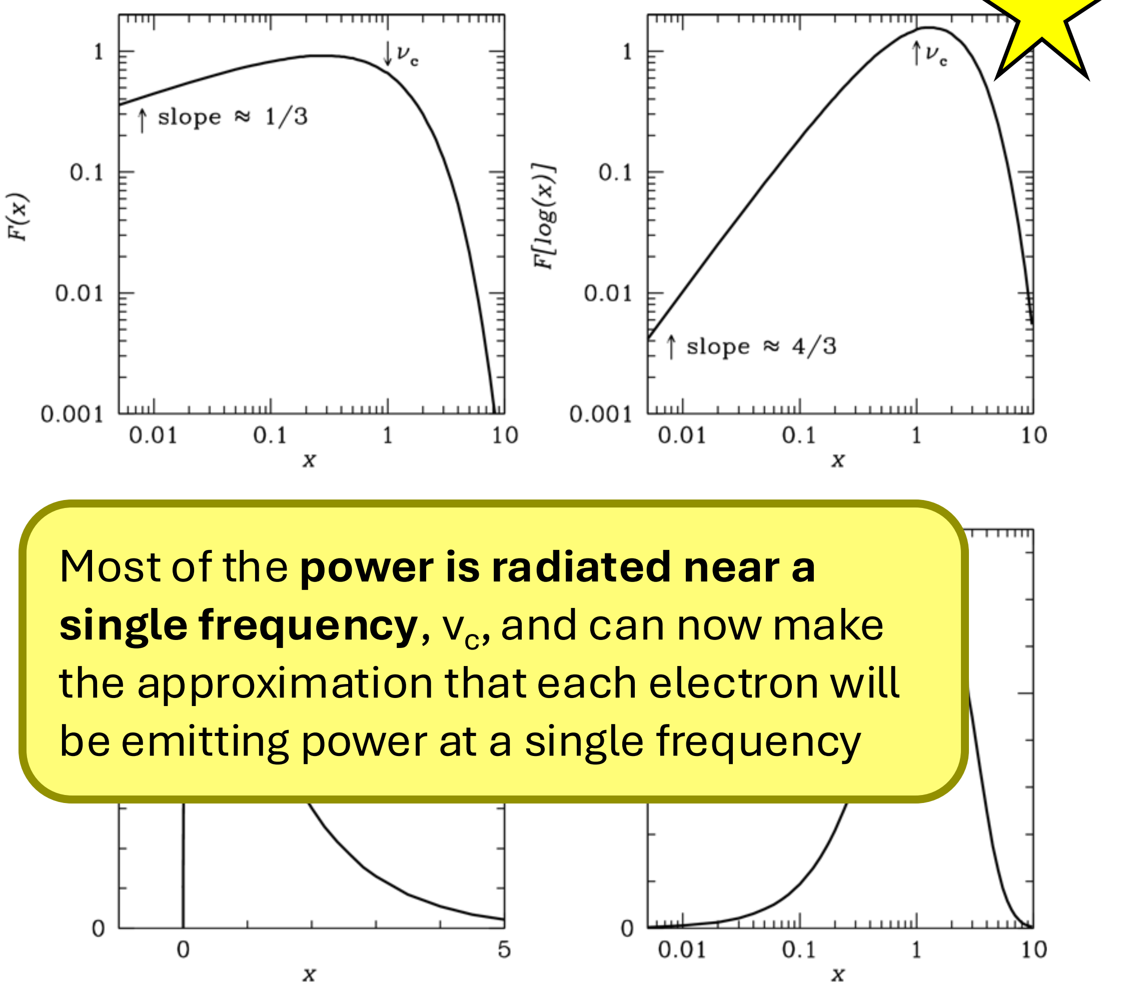
And the **formal solution to the power spectrum** is,

$$P(\nu) = \frac{\sqrt{3}e^3 B \sin\alpha}{m_e c^2}\left(\frac{\nu}{\nu_c}\right)\int_{\nu/\nu_c}^{\infty} K_{5/3}(\eta)\, d\eta, \qquad (5.66)$$

Where,

$$\nu_c = \frac{3}{2}\gamma^2\nu_G \sin\alpha. \qquad (5.67)$$

Fig. 5.6 (ERA) Four different ways to plot the synchrotron spectrum of a single electron:



Most of the **power is radiated near a single frequency**, ν_c, and can now make the approximation that each electron will be emitting power at a single frequency

Synchrotron Radiation (ERA Chapter 5)

Synchrotron Spectra (5.3)

(5.3.1) Synchrotron Spectrum of a Single Electron

So, what does the spectrum actually look like?

The synchrotron spectrum of a single electron is fairly flat at low frequencies and tapers off at frequencies above,

$$\nu_{\max} \approx \frac{1}{2\Delta t_{\rm p}} \approx \pi\gamma^2\nu_{\rm G}\sin\alpha \propto \gamma^2 B_\perp. \qquad (5.65)$$

The continuous distribution looks more like a power-law that should be plotted in log space →

And the **formal solution to the power spectrum** is,

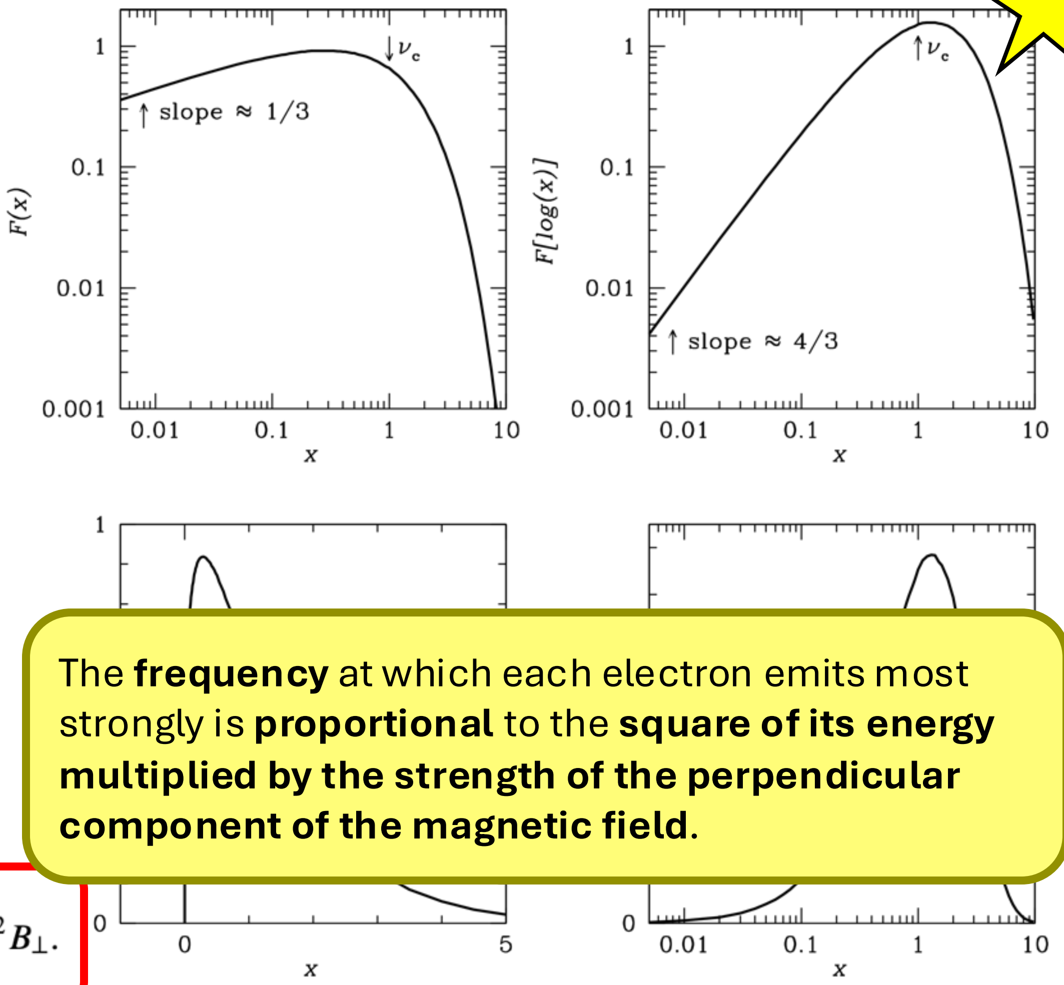
$$P(\nu) = \frac{\sqrt{3}e^3 B\sin\alpha}{m_{\rm e}c^2}\left(\frac{\nu}{\nu_{\rm c}}\right)\int_{\nu/\nu_{\rm c}}^{\infty} K_{5/3}(\eta)\, d\eta, \qquad (5.66)$$

Where,

$$\nu_{\rm c} = \frac{3}{2}\gamma^2\nu_{\rm G}\sin\alpha. = \left(\frac{3}{2}\sin\alpha\right)\left(\frac{E}{mc^2}\right)^2\frac{eB}{2\pi m_{\rm e}c} \propto E^2 B_\perp.$$

Four different ways to plot the synchrotron spectrum of a single electron:

Fig. 5.6 (ERA)



The **frequency** at which each electron emits most strongly is **proportional** to the **square of its energy multiplied by the strength of the perpendicular component of the magnetic field**.

Synchrotron Radiation (ERA Chapter 5)

Synchrotron Spectra (5.3)

(5.3.2) ***Synchrotron Spectra of Optically Thin Radio Sources***

Most astrophysical sources of synchrotron radiation behave as power laws and have spectral indices near $\alpha \sim 0.75$ ($\delta \sim 2.5$) that reflects electron energy distributions

As we did for free-free, now we can write the **emission coefficient j_ν** for an ensemble of electrons where 'δ' is now used for our power law that describes the **number of electrons per unit volume**,

$$n(E)\,dE \propto E^{-\delta} dE, \qquad (5.70)$$

$$j_\nu d\nu = -\frac{dE}{dt} n(E)\,dE, \qquad (5.73)$$

Lots of substituting later (see text) we have,

$$\boxed{j_\nu \propto B^{(\delta+1)/2} \nu^{(1-\delta)/2}.} \qquad (5.79)$$

Synchrotron Radiation (ERA Chapter 5)

Synchrotron Spectra (5.3)

*(5.3.2) **Synchrotron Spectra of Optically Thin Radio Sources***

Most astrophysical sources of synchrotron radiation behave as power laws and have spectral indices near α ~ 0.75 (δ ~ 2.5) that reflects electron energy distributions

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$$j_\nu \propto B^{(\delta+1)/2} \nu^{(1-\delta)/2}. \qquad (5.78)$$

The spectral index α=−dlnS/dlnν depends only on δ:

$$\alpha = \frac{\delta - 1}{2}. \qquad (5.79)$$

Synchrotron Radiation (ERA Chapter 5)

Synchrotron Spectra (5.3)

(5.3.2) ***Synchrotron Spectra of Optically Thin Radio Sources***

Most astrophysical sources of synchrotron radiation behave as power laws and have spectral indices near α ~ 0.75 (**δ ~ 2.5**) that reflects electron energy distributions

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Lots of substituting later (see text) we have,

$$j_\nu \propto B^{(\delta+1)/2} \nu^{(1-\delta)/2}. \qquad (5.78)$$

The rate at which an electron loses energy to synchrotron radiation is proportional to E^2 and therefore higher energy electrons are depleted more rapidly AND as we just saw, the critical frequency is also proportional to E^2 so the **source spectra steepens at high frequencies!**

The spectral index α=−dlnS/dlnν depends only on δ:

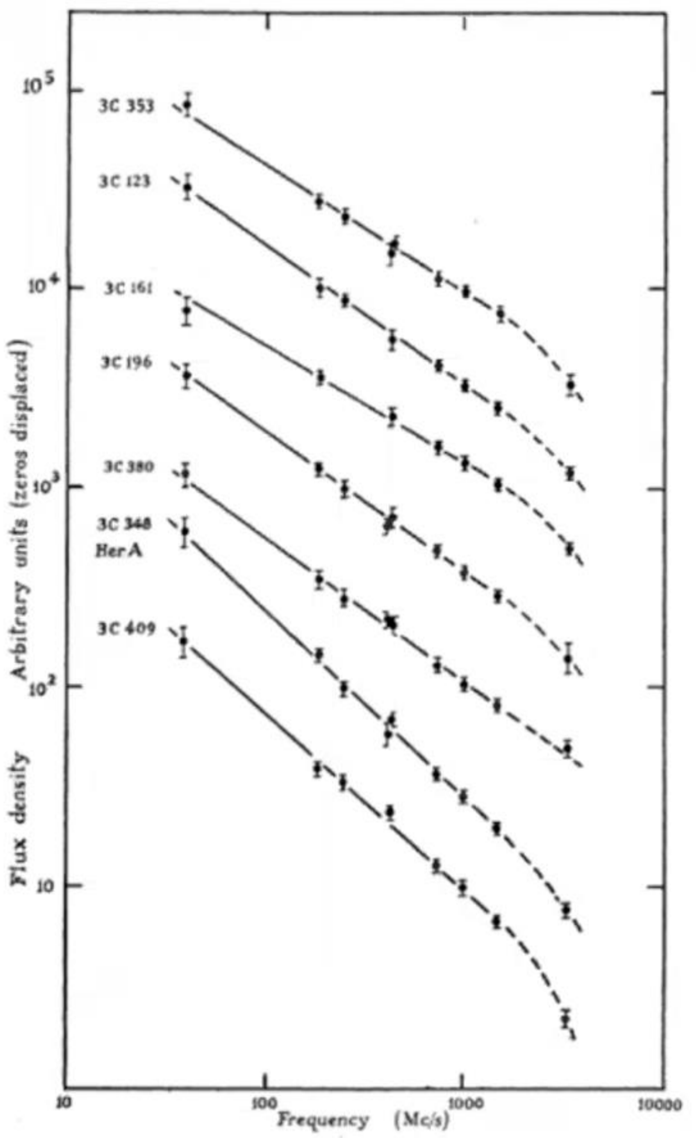
$$\alpha = \frac{\delta - 1}{2}. \qquad (5.79)$$

Synchrotron Radiation (ERA Chapter 5)

Synchrotron Spectra (5.3)

Spectrum of radio galaxies that are→ strong synchrotron emitters

Power-law evident here, with some mild differences which is determined by relativistic electron distribution



Slide Credit: Jim Braatz

Synchrotron Radiation (ERA Chapter 5)

Synchrotron Spectra (5.3)

(5.3.3) ***Synchrotron Self-Absorption***

Here **we discuss line brightness and fluxes**…

Even if the ensemble of electrons has a nonthermal energy distribution, we still define an **'effective temperature'** of relativistic electrons:

$$T_e \equiv \frac{E}{3k} = \frac{\gamma m_e c^2}{3k}, \quad (5.83)$$

Removing γ and solving numerically we have,

$$\left(\frac{T_e}{\mathrm{K}}\right) \approx 1.18 \times 10^6 \left(\frac{\nu}{\mathrm{Hz}}\right)^{1/2} \left(\frac{B}{\mathrm{gauss}}\right)^{-1/2}. \quad (5.85)$$

→ Relativistic electrons emitting synchrotron radiation at ν = 0.1 GHz =10^8 Hz in a B=100μgauss = 10^{-4} gauss magnetic field is T_e ~ 10^{12} K

Synchrotron Radiation (ERA Chapter 5)

Synchrotron Spectra (5.3)

(5.3.3) ***Synchrotron Self-Absorption***

At **low frequencies**, the **brightness temperature approaches the effective temperature**, the source become optically thick to synchrotron self-absorption.

We are in the Rayleigh Jeans limit so,

$$I_\nu \approx \frac{2kT_e\nu^2}{c^2} \propto \nu^{1/2}\nu^2 B^{-1/2} = \nu^{5/2}B^{-1/2}. \quad (5.88)$$

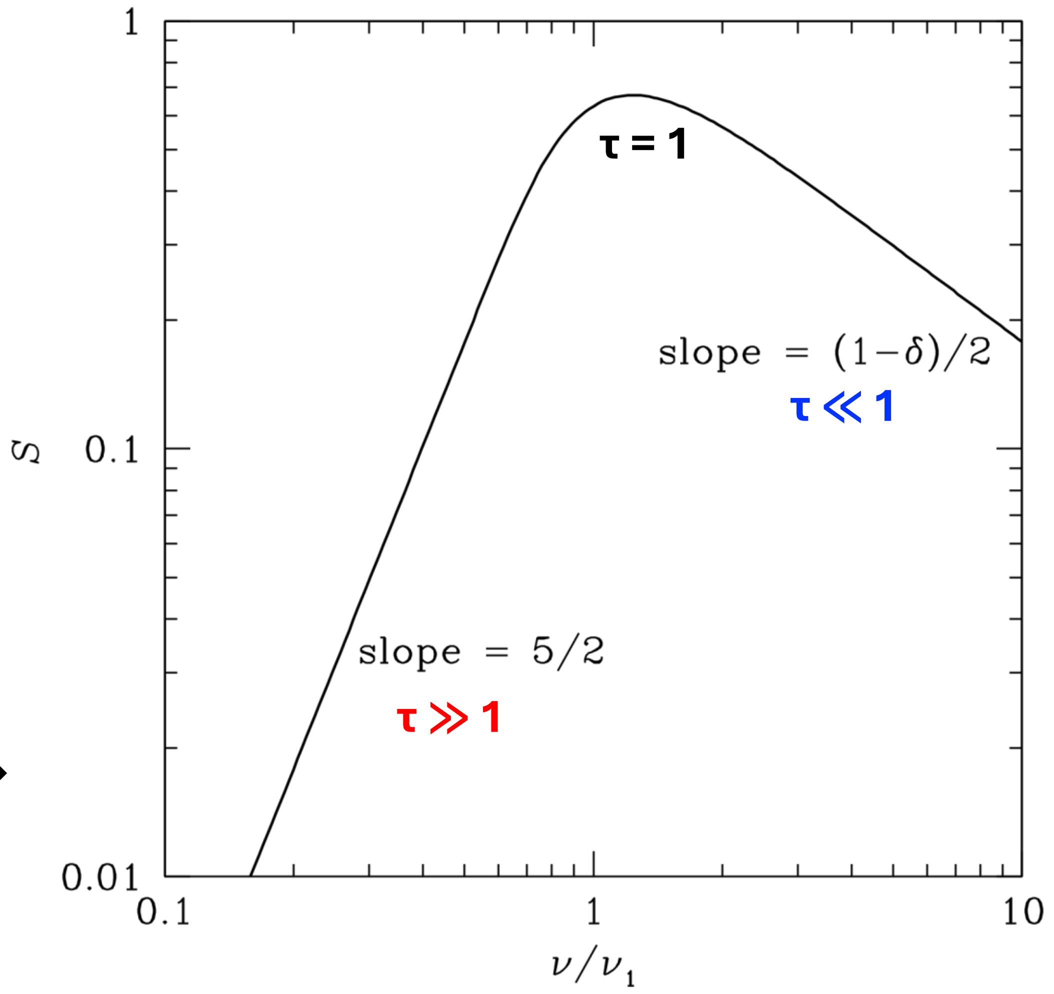
Where, $\boxed{S(\nu) \propto \nu^{5/2},}$ (5.89)

The full spectrum of a homogeneous cylindrical synchrotron source →

$$S \propto \left(\frac{\nu}{\nu_1}\right)^{5/2}\left\{1 - \exp\left[-\left(\frac{\nu}{\nu_1}\right)^{-(\delta+4)/2}\right]\right\}, \quad (5.90)$$

Fig. 5.7 (ERA)

Idealized case:



Synchrotron Radiation (ERA Chapter 5)

Synchrotron Spectra (5.3)

(5.3.3) ***Synchrotron Self-Absorption***

At **low frequencies**, the **brightness temperature approaches the effective temperature**, the source become optically thick to synchrotron self-absorption.

$$\left(\frac{T_e}{\mathrm{K}}\right) \approx 1.18 \times 10^6 \left(\frac{\nu}{\mathrm{Hz}}\right)^{1/2} \left(\frac{B}{\mathrm{gauss}}\right)^{-1/2}. \quad (5.85)$$

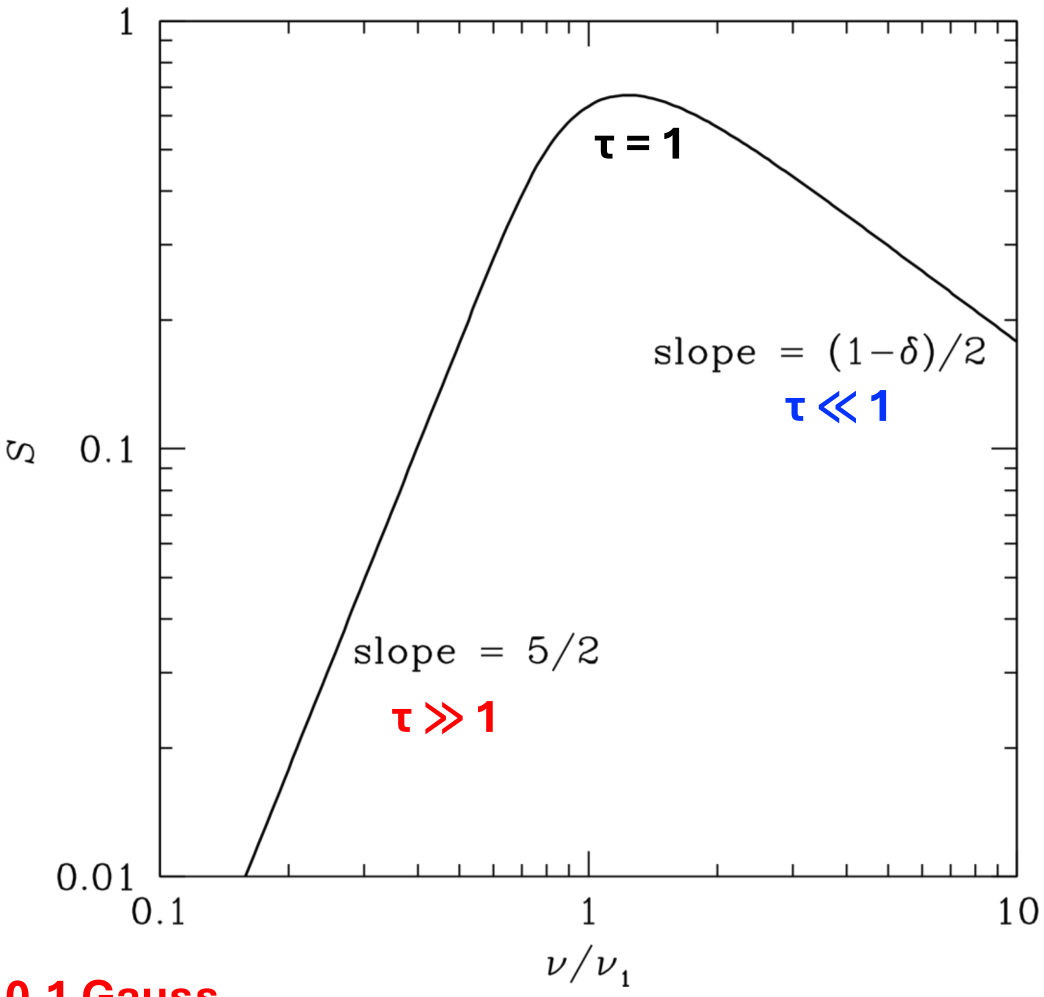
Can now estimate of the **magnetic field strength**

$$\left(\frac{B}{\mathrm{gauss}}\right) \approx 1.4 \times 10^{12} \left(\frac{\nu}{\mathrm{Hz}}\right) \left(\frac{T_b}{\mathrm{K}}\right)^{-2}. \quad (5.91)$$

→ $T_e = T_b \approx 10^{11}$ K at v=1 GHz has a magnetic field strength of 0.1 Gauss

Fig. 5.7 (ERA)

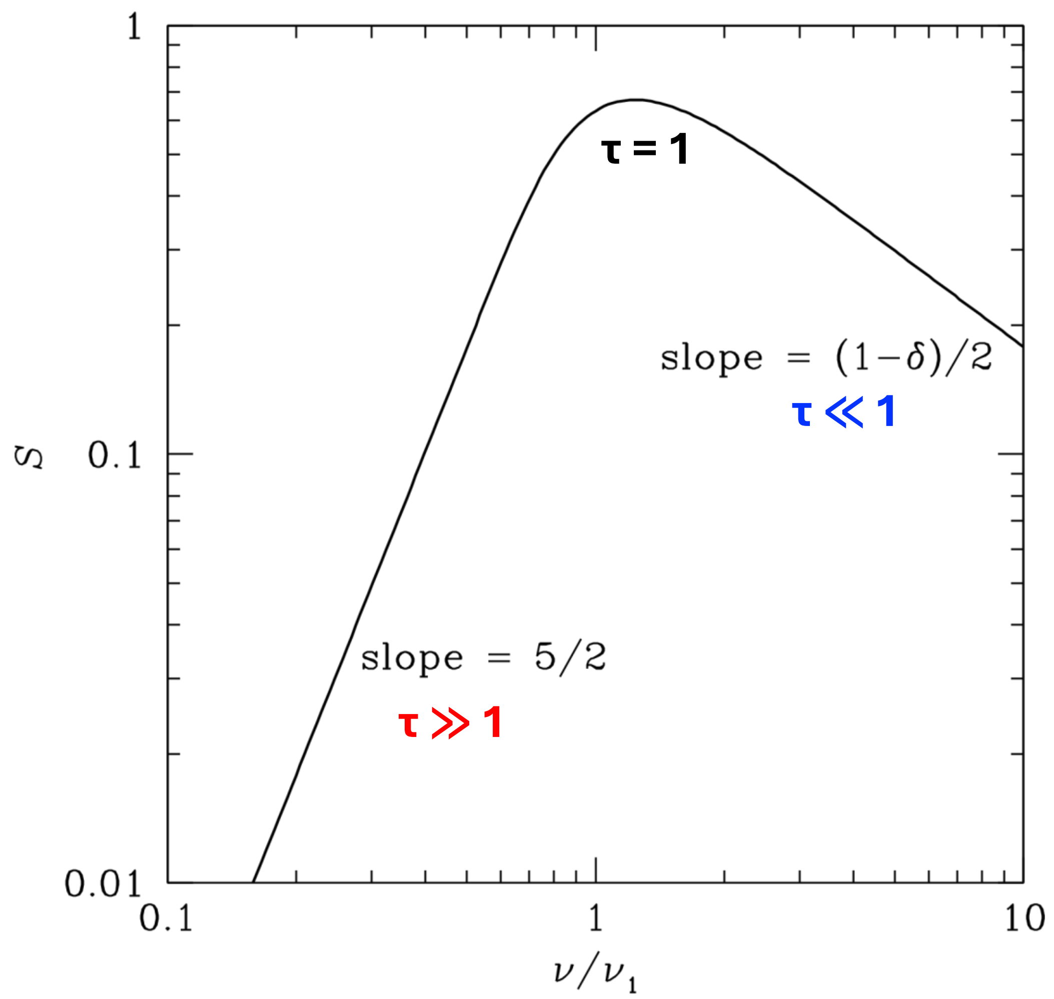
Idealized case:



Synchrotron Radiation

Fig. 5.7 (ERA)

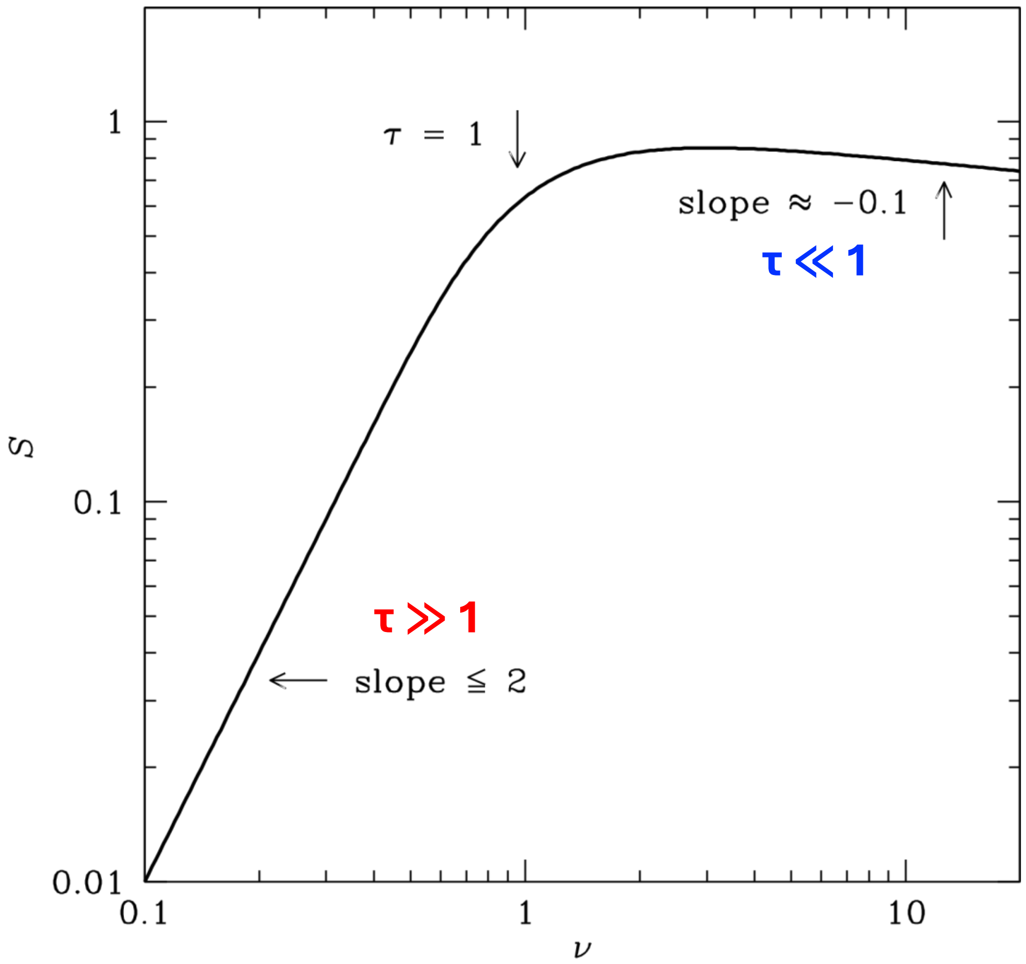
Idealized case:



Free-Free Radiation

Fig. 4.8 (ERA)

Idealized case:



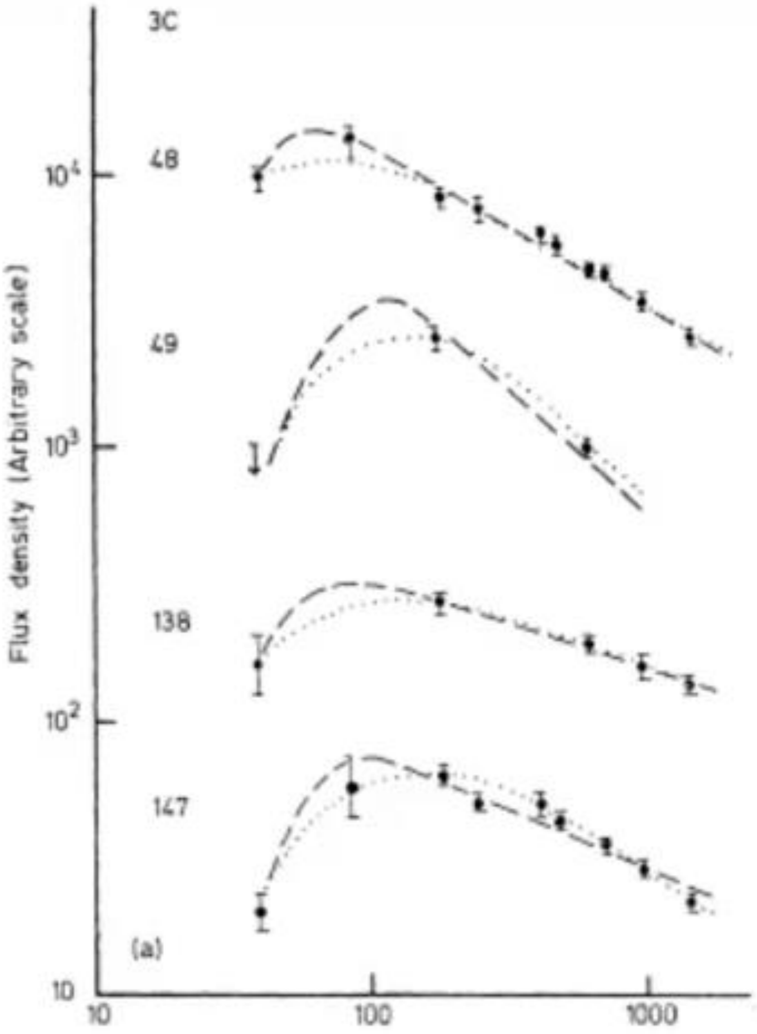
Synchrotron Radiation (ERA Chapter 5)

Synchrotron Spectra (5.3)

(5.3.3) ***Synchrotron Self-Absorption***

Spectrum of radio galaxies that are→ strong synchrotron emitters

Turn over or 'drop-off' at low frequency that shows the ***Synchrotron Self-Absorption***



Slide Credit: Jim Braatz

National Radio Astronomy Observatory

Synchrotron Radiation (ERA Chapter 5)

Synchrotron Spectra (5.3)

(5.3.3) ***Synchrotron Self-Absorption***

Representative spectra of radio galaxies → and quasars show diversity in nonuniform magnetic fields and electron energy distributions in geometrically complex structures

Fig. 5.8 (ERA)

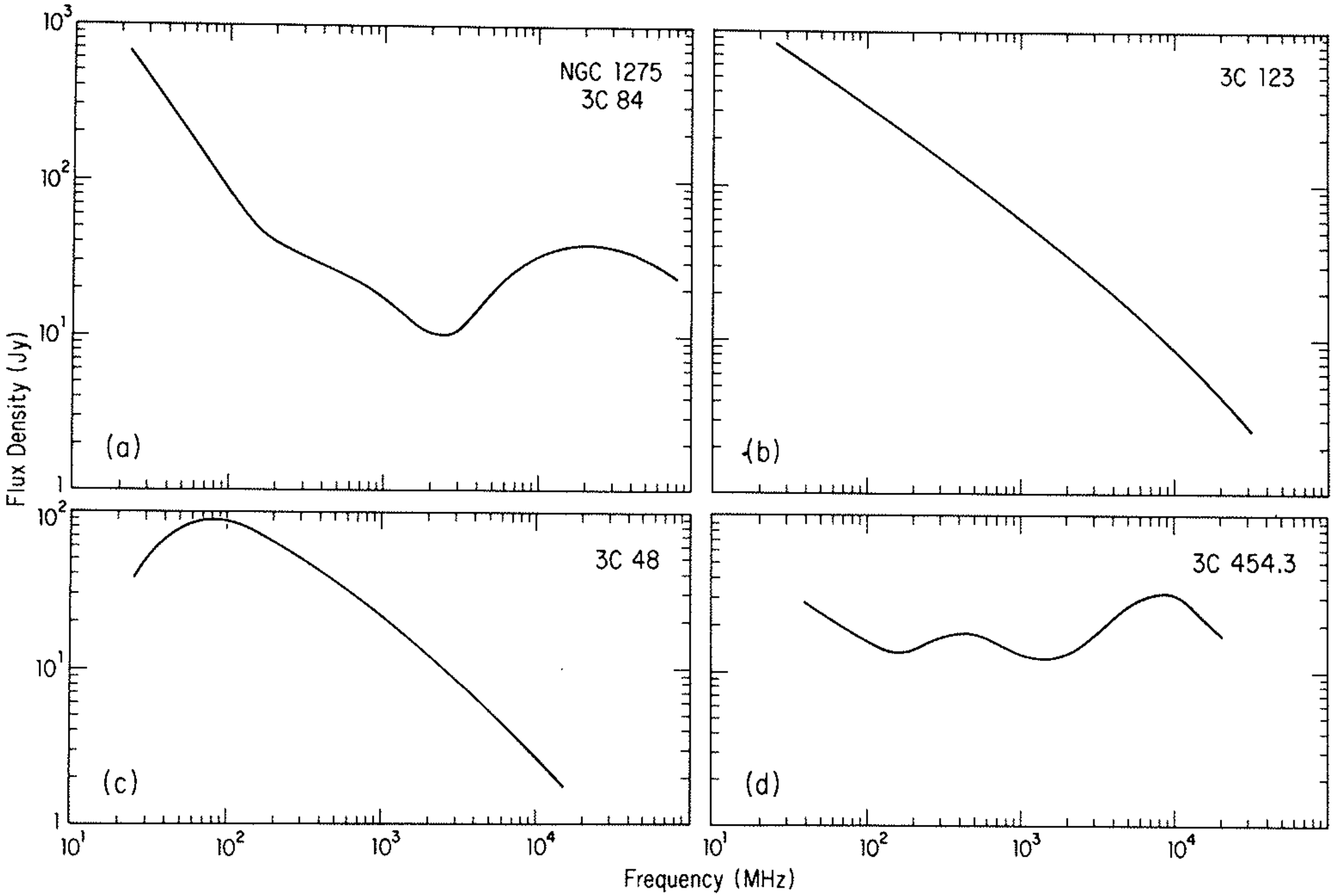


Fig. 2.24 (ERA)

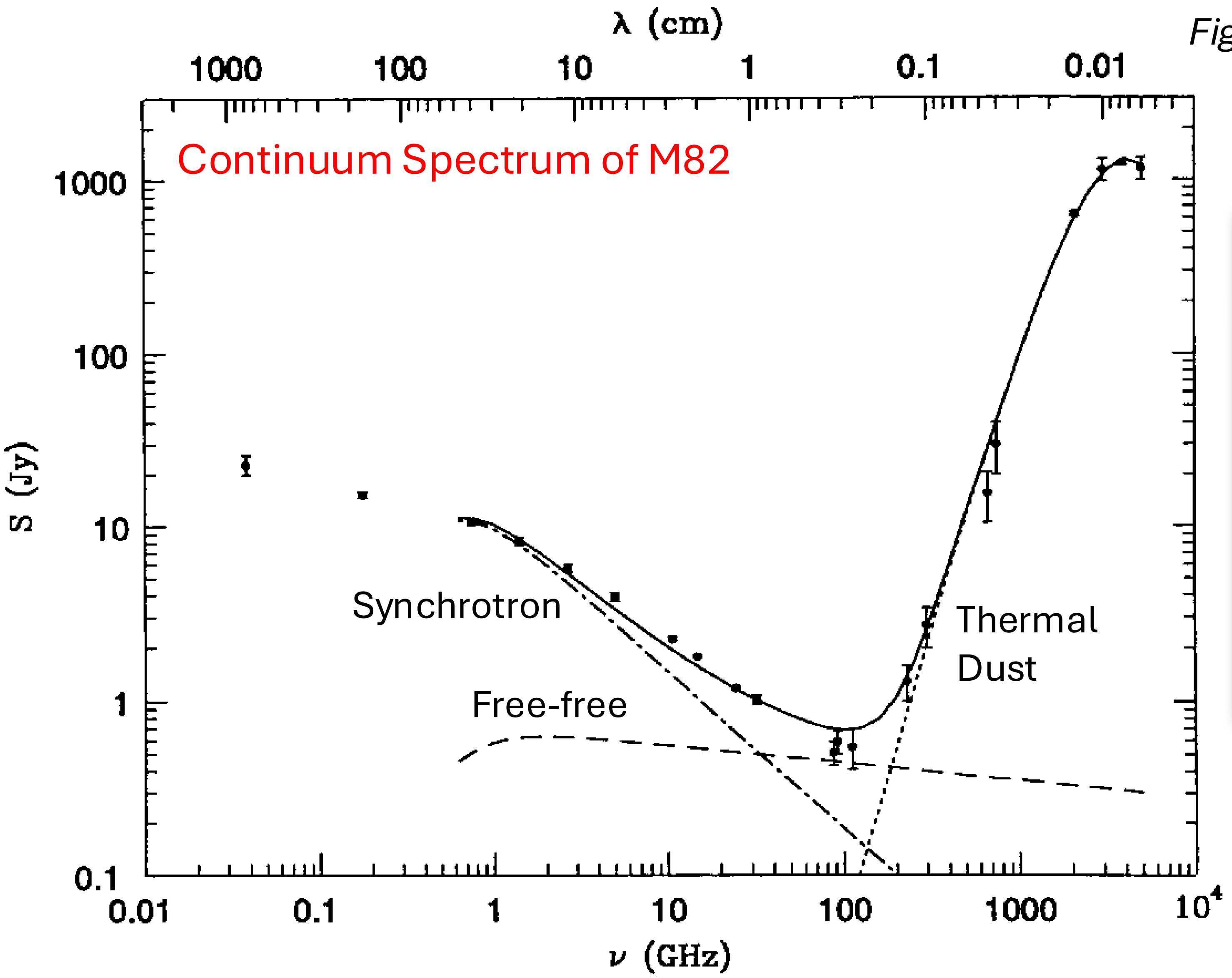


Fig. 8.13 (ERA) Radio continuum emission from M82. Image credit: Josh Marvil (NM Tech/NRAO), Bill Saxton (NRAO/AUI/NSF), Hubble (NASA/ESA/STScI).

Synchrotron Radiation (ERA Chapter 5)

Synchrotron Sources (5.4)

(5.4.1) Minimum Energy and Equipartition

Idea here to find the minimum total energy in relativistic particles and magnetic fields required to produced a synchrotron source of a certain radio luminosity

The text goes over derivations to solve for **electron energy density** by integrating over the number density of electrons *n(E)dE* in the energy range *E* to *E* + *dE* times electrons with energy, *E*.
We get:

$$\boxed{U_{\rm e} \propto B^{-3/2},} \quad (5.98)$$

and a total energy density (all cosmic rays) of $U_{\rm E} = (1+\eta)\, U_{\rm e}$

Where we consider the "invisible" cosmic ray protons and heavier ions because they still contribute to the total cosmic-ray particle energy, where η is the ion/electron energy ratio

Combining the magnetic energy density, $U_B \propto B^2.$ (5.99)

The total energy is: $\boxed{U = (1+\eta)\, U_{\rm e} + U_B.}$ (5.100)

Synchrotron Radiation (ERA Chapter 5)

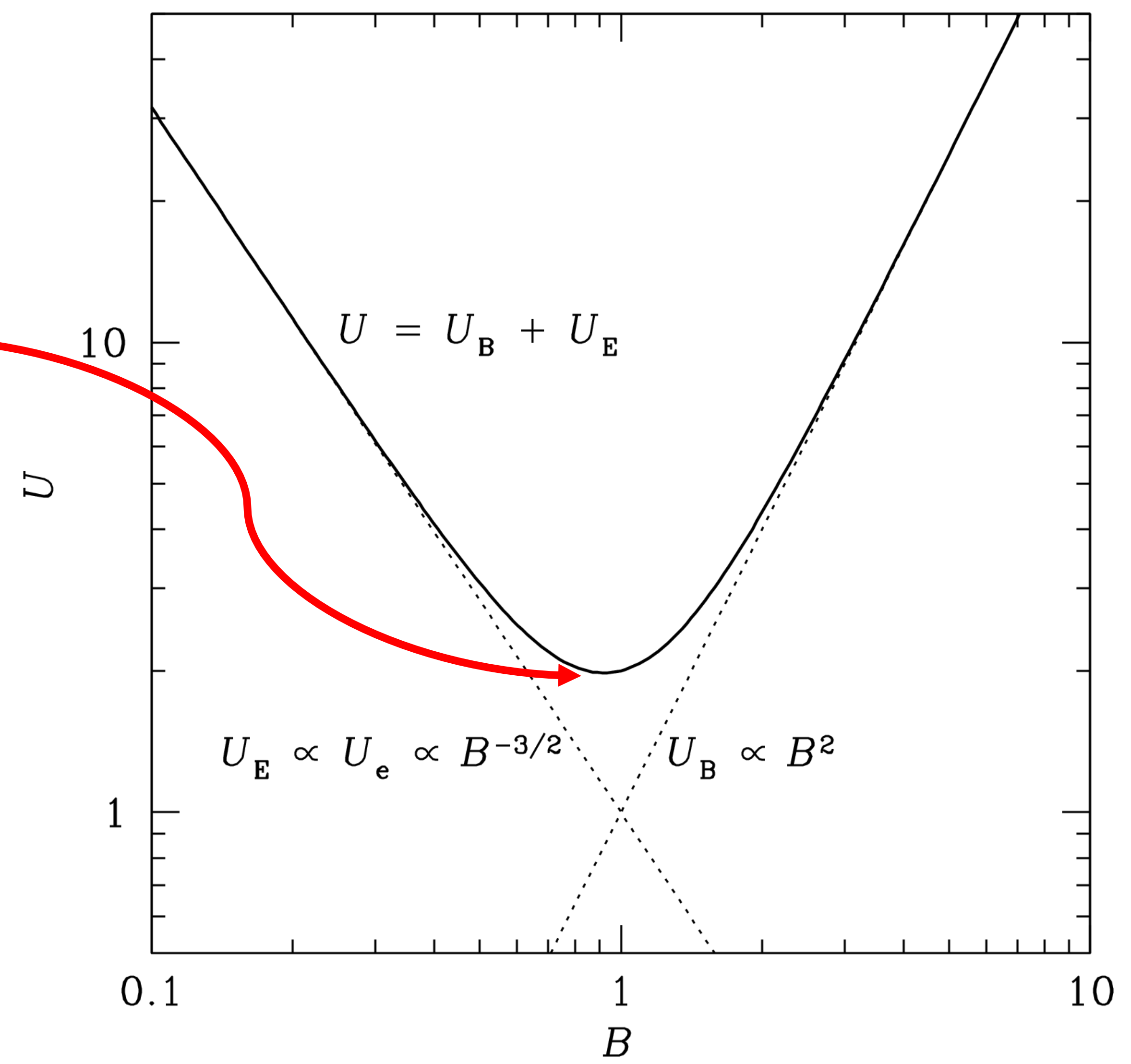
Synchrotron Sources (5.4)

(5.4.1) ***Minimum Energy and Equipartition***

Main Takeaway:

There are greatly differing dependences of U_e and U_b on '*B*' so the total energy density E has a fairly sharp minimum near equipartition, i.e., the point at which $(1+\eta)U_e \approx U_B$

Fig. 5.9 (ERA)



Synchrotron Radiation (ERA Chapter 5)

Synchrotron Sources (5.4)

(5.4.1) ***Minimum Energy and Equipartition***

Main Takeaway:

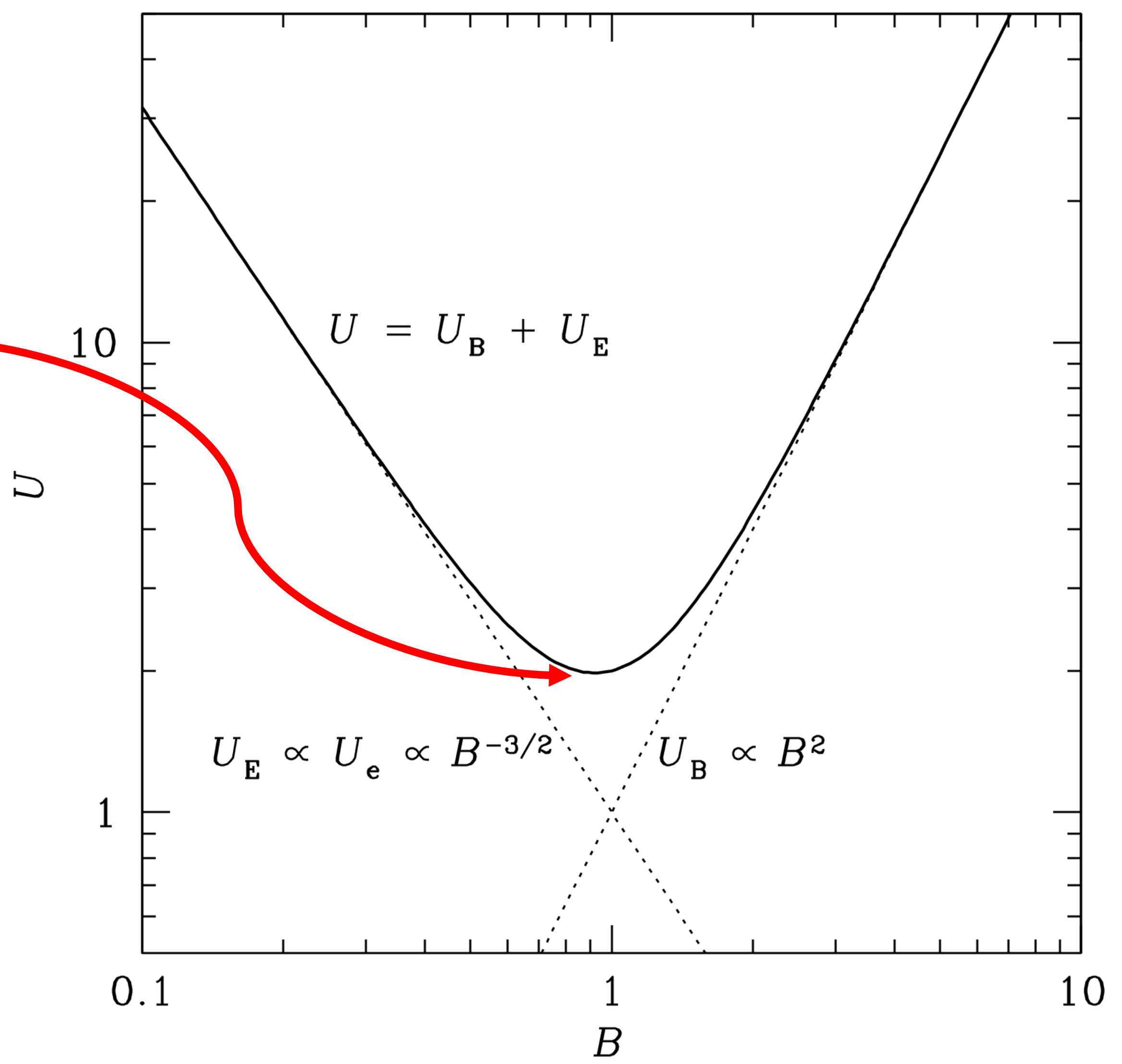
There are greatly differing dependences of U_e and U_b on '*B*' so the total energy density E has a fairly sharp minimum near equipartition, i.e., the point at which $(1+\eta)U_e \approx U_B$

The ratio of cosmic-ray particle energy density to magnetic field energy that minimizes the total energy is,

$$\frac{\text{particle energy density}}{\text{magnetic field energy density}} = \frac{(1+\eta)\,U_e}{U_B} = \frac{4}{3}. \quad (5.107)$$

Nearly unity!

Fig. 5.9 (ERA)



Synchrotron Radiation (ERA Chapter 5)

Synchrotron Sources (5.4)

(5.4.1) ***Minimum Energy and Equipartition***

Main goal then is to extract out the minimum-energy magnetic field strength for a source of radio luminosity *L* and radius *R*,

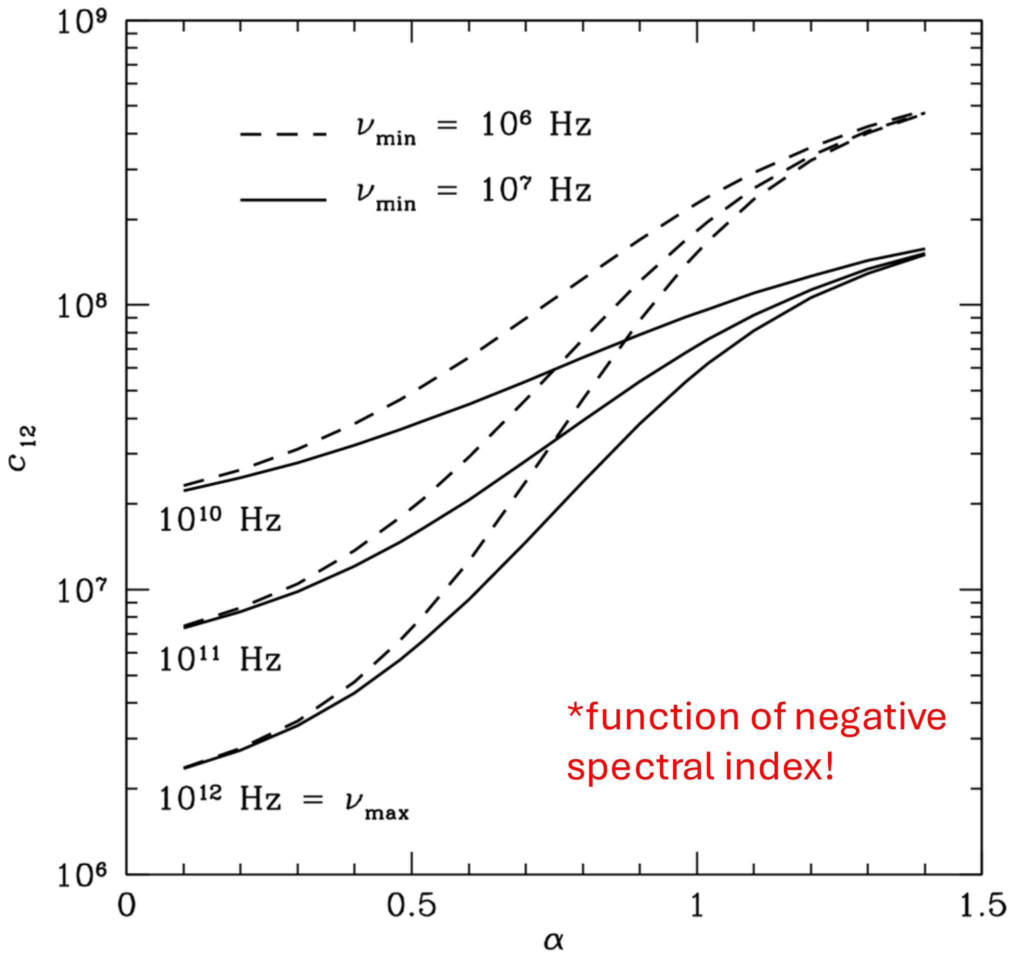
$$B_{\rm min} = [4.5\,(1+\eta)\,c_{12} L]^{2/7} R^{-6/7} \quad \text{gauss} \qquad (5.109)$$

And the corresponding total energy,

$$E_{\rm min}\,(\text{total}) = c_{13}\,[(1+\eta)\,L]^{4/7} R^{9/7} \quad \text{ergs.} \qquad (5.110)$$

Which have been simplified numerically (see text and references Wilson et al., and Pacholczyk).

Fig. 5.10 (ERA)



Synchrotron Radiation (ERA Chapter 5)

Synchrotron Sources (5.4)

(5.4.1) ***Minimum Energy and Equipartition***

Another key term to know is the **synchrotron lifetime,** defined as the ratio of the total electron energy E_e to the energy loss rate in terms of luminosity L:

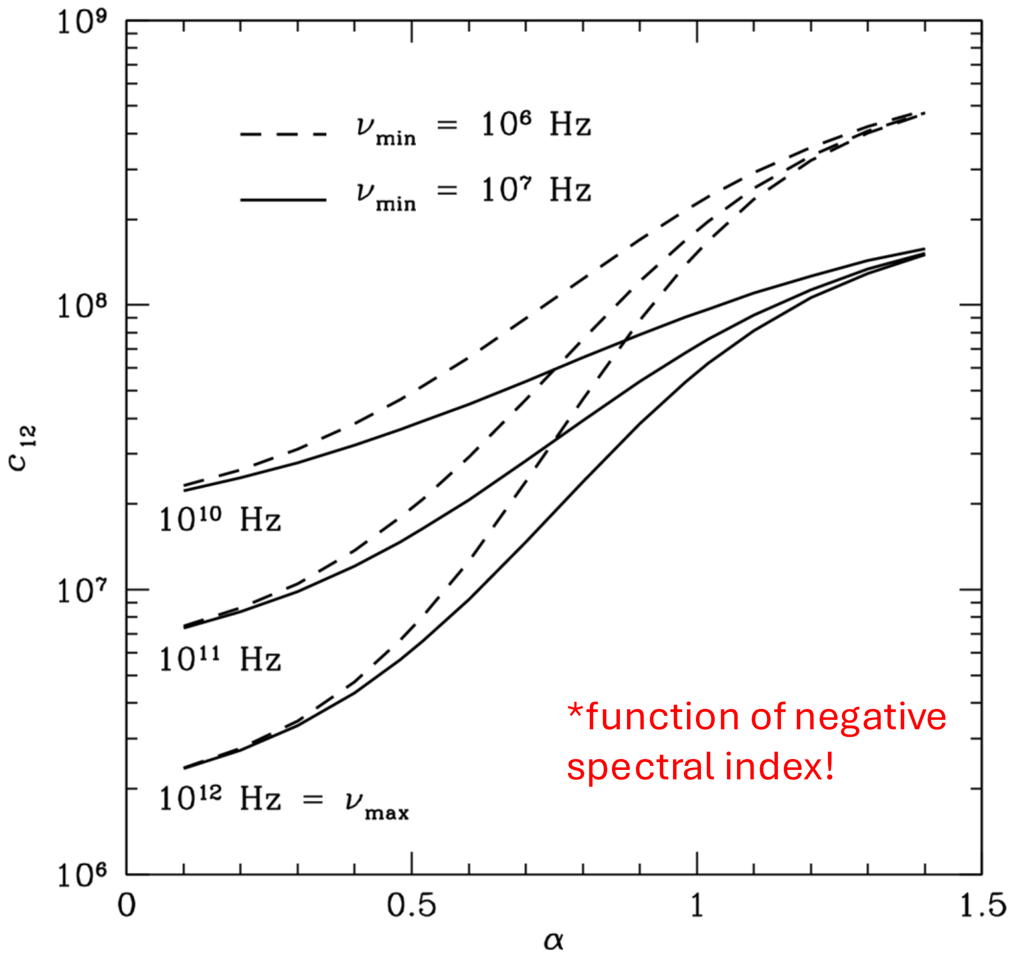
$$\tau_s \equiv \frac{E_e}{L}. \qquad (5.111)$$

If other loss mechanisms (e.g., inverse-Compton scattering) are significant, the **actual source lifetime will be shortened**

And can be written in terms of c_{12} and B-field:

$$\boxed{\tau_s \approx c_{12} B_{\perp}^{-3/2}.} \qquad (5.112)$$

Fig. 5.10 (ERA)



Synchrotron Radiation (ERA Chapter 5)

Synchrotron Sources (5.4)

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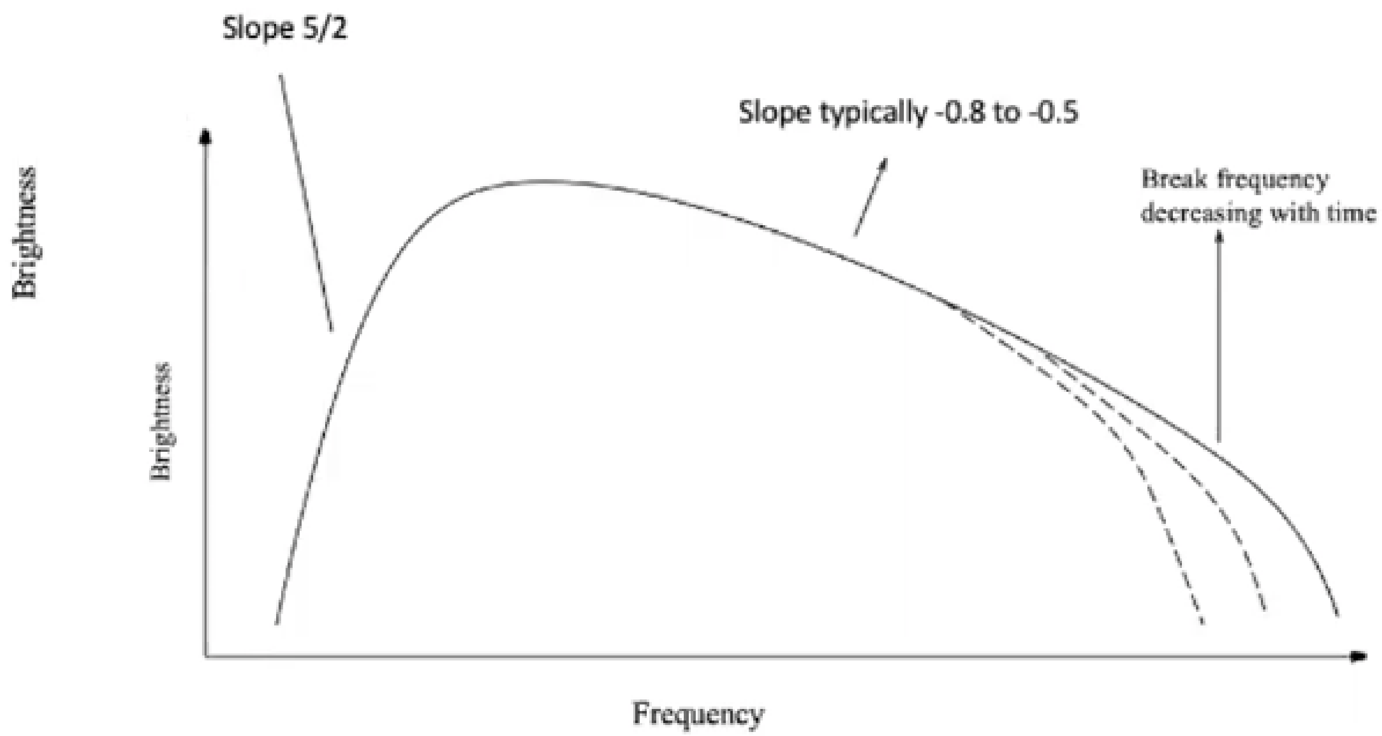

Slide Credit: Jim Braatz